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RESEARCH ARTICLE OPEN ACCESS

NONLINEAR SOLITON INTERACTIONS IN RELATIVISTIC MAGNETIZED SYSTEMS: IMPLICATIONS FOR SPACE AND ASTROPHYSICAL PHENOMENA

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ABSTRACT

This paper explores the formation and propagation of solitons in a three-fluid weakly relativistic plasma system influenced by an external magnetic field. Employing Normal Mode Analysis and the Reductive Perturbation Technique (RPT), we derive phase velocity relations and examine the effects of density inhomogeneity, relativistic corrections, and magnetic field strength on nonlinear wave evolution [1-3]. Additionally, computational models are utilized to optimize soliton stability predictions and facilitate real-time plasma diagnostics, enhancing their applicability in space propulsion systems and astrophysical environments [4-6]. The study reveals the excitation of a new medium mode due to plasma inhomogeneity, which significantly affects soliton dynamics, particularly in magnetized astrophysical plasmas and controlled nuclear fusion experiments [7-9]. Our results provide critical insights into next-generation plasma-based propulsion technologies and nonlinear wave interactions in extreme astrophysical conditions [10-12].

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INTRODUCTION

The study of nonlinear wave propagation in plasma is an essential aspect of modern plasma physics, with direct implications for space propulsion and high-energy astrophysics [13-15]. Solitons, which are self-reinforcing wave packets that maintain their shape while propagating, play a crucial role in understanding nonlinear plasma dynamics. Incorporating relativistic effects, external magnetic fields, and density inhomogeneity significantly complicates soliton behavior, making their study highly relevant for advanced plasma-based technologies [16-18].Recent advancements in plasma modeling have demonstrated the potential of machine learning algorithms to improve soliton predictions, energy confinement, and real-time plasma diagnostics [19]. Multi-fluid plasma systems are fundamental in understanding astrophysical phenomena such as pulsar magnetospheres, accretion disks, and active galactic nuclei [20-21]. Moreover, optimized plasma propulsion mechanisms are currently being explored as alternatives to traditional chemical propulsion systems for deep-space missions [22].Recent studies indicate that density gradients can excite additional wave modes in plasmas, leading to new nonlinear structures and soliton formations [23].

These effects are particularly relevant in pulsar magnetospheres, black hole accretion disks, and early universe conditions [24-25]. The motivation of this work is to examine how relativistic effects, plasma inhomogeneity, and external magnetic fields influence soliton propagation in three-fluid plasmas [26-27]. We consider a three-fluid plasma system consisting of electrons, positrons, and ions immersed in an external magnetic field. The governing equations include the continuity equations, momentum balance equations, and Poisson's equation [28]. Using the Reductive Perturbation Technique (RPT), we derive a modified Korteweg–de Vries (KdV) equation that accounts for relativistic and inhomogeneity-induced nonlinear effects [29].The incorporation of computational optimization techniques enables improved accuracy in predicting soliton characteristics, including amplitude, width, and velocity. Neural networks trained on experimental and simulated plasma datasets refine soliton parameter estimation, reducing computational overhead and improving model accuracy [30-32].

Basic equations and Normal Mode Analysis using Lorentz force: In this section, in the positive ion, positron and electron fluid equations the Lorentz force contribution will be taken into account. A finite amplitude ion accoustic wave propogation in a three fluid homogeneous plasma having finite temperature weakly relativistic ions, electrons and positrons is considered under the effect of a uniform external magnetic field $\vec{B}=B_o\hat{y}$. The wave propagation is considered to be in the (x, z) Plane at an angle with the direction of magnetic field kinetic effects such as Landau damping, heat conduction, viscosity etc are neglected. Under such conditions, the normalized form of continuity and momentum equations for ion, electron and positron fluids and the Poisson's equation is used to study three fluid homogeneous magnetized plasma model.

For ion fluid.

Continuity equation is given

$$\frac{\partial n_i}{\partial t}+\nabla(n_i u)=0 \qquad \text{.....(1)}$$

using lorentz force contribution

$$\vec{F}=q(\vec{E}+\vec{u}\times\vec{B}) \qquad \text{.....(2)}$$

We are considering magnetic field in y - direction, hence velocity component in y - direction will be neglected. In the above equation q is charge on positive ion particle, $\vec{u}$ is velocity of positive ion particle and is the magnetic field in y - direction.

So, applying equation (2) in equation (1), we get

$$\frac{\partial n_i}{\partial t}+\frac{\partial(n_i u_x)}{\partial x}+\frac{\partial(n_i u_z)}{\partial z}=0.....[3]$$

$$n_i\frac{\partial(y_i u_x)}{\partial t}+n_i u_x\frac{\partial(y_i u_x)}{\partial x}+n_i u_z\frac{\partial(y_i u_x)}{\partial z}+n_i\frac{\partial\phi}{\partial x}-an_i u_y+2\sigma\frac{\partial n_i}{\partial x}=0.....[4]$$

using Lorentz force contribution in momentum equation of ion we get.

$$\frac{\partial n_y}{\partial t}+u_x\frac{\partial u_y}{\partial x}+u_z\frac{\partial u_y}{\partial z}+au_x=0.....[5]$$

$$n_i\frac{\partial u_y}{\partial t}+n_i u_x\frac{\partial u_x}{\partial x}+n_i u_z\frac{\partial u_z}{\partial z}+n_i\frac{\partial\phi}{\partial z}+2\sigma\frac{\partial n_i}{\partial z}=0 \qquad \text{.....(6)}$$

For electron fluid

Continuity equation is given as

$$\frac{\partial n_e}{\partial t}+\nabla.(n_e v)=0.....[7]$$

Using Lorentz force contribution :

$$\vec{F}=e(\vec{E}+\vec{v}\times\vec{B}) \qquad \text{.....(8)}$$

We are considering magnetic field in y - direction, hence velocity componend in y - direction will be neglected. In the above equation (8), e is charge on electron particle, $\vec{v}$ is velocity of electron & $\vec{B}=B_o\hat{y}$. is the magnetic field in y - direction. So applying equation (8) in equation (7) we get

$$\frac{\partial n_e}{\partial t}+\frac{\partial(n_e v_x)}{\partial x}+\frac{\partial(n_e v_z)}{\partial z}=0 \qquad \text{.....(9)}$$

$$n_e\frac{\partial(y_e v_x)}{\partial t}+n_e v_x\frac{\partial(\gamma_e v_x)}{\partial x}+n_e v_z\frac{\partial(\gamma_e v_x)}{\partial z}-\frac{m_i}{m_e}n_e\frac{\partial\varphi}{\partial x}+\frac{m_i}{m_e}an_e v_y+\frac{m_i}{m_e}\frac{\partial n_e}{\partial x}=0 \qquad \text{.....(10)}$$

Now using Lorentz force contribution in momentum equation for electron we get.

$$\frac{\partial v_y}{\partial t} + v_x \frac{\partial v_y}{\partial x} + v_z \frac{\partial v_y}{\partial z} - \frac{m_i}{m_e} a v_x = 0 \quad \text{.....(11)}$$

$$n_e \frac{\partial v_z}{\partial t} + n_e v_x \frac{\partial v_z}{\partial x} + n_e v_z \frac{\partial v_z}{\partial z} - n_e \frac{m_i}{m_e} \frac{\partial \varphi}{\partial z} + \frac{m_i}{m_e} \frac{\partial n_e}{\partial z} a v_x = 0 \quad \text{.....(12)}$$

For positron fluid:

Continuity equation is given by :

$$\frac{\partial n_p}{\partial t} + \nabla . \left(n_p w \right) = 0 \quad \text{.....(13)}$$

Using lorentz force contribution

$$\vec{F} = e\left(\vec{E} + \vec{w} \times \vec{B} \right) \quad \text{.....(14)}$$

In the above equation (14) we are considering magnetic field in y - direction, hence velocity component in y - direction will be reglected and q, e is the charge on positron, $\vec{w}$ is velocity of positron and $\vec{B}=\vec{B}_o\hat{y}$ is the magnitude field in y- firection. So applying equation (14) in equation (13) we get

$$\frac{\partial n_p}{\partial t} + \frac{\partial (n_p w_x)}{\partial x} + \frac{\partial (n_p w_2)}{\partial z} = 0$$

$$\frac{n_p \partial(y_p w)}{\partial t} + n_p w_x \frac{\partial(y_p w_x)}{\partial x} + n_p w_z \frac{\partial(\gamma_p w_z)}{\partial z} - \frac{m_i}{m_p} n_p \frac{\partial \varphi}{\partial x} + \frac{m_i}{m_p} a n_p w_y + \frac{m_i}{m_p} \frac{\partial n_p}{\partial x} = 0 \quad \text{.....(15)}$$

Now using Lorentz force contribution in momentum equation of positron we get.

$$\frac{\partial w_y}{\partial t} + w_x \frac{\partial w_y}{\partial x} + w_z \frac{\partial w_y}{\partial z} - \frac{m_i}{m_p} a w_x = 0 \quad \text{.....(16)}$$

$$n_p \frac{\partial w_z}{\partial t} + n_p w_x \frac{\partial w_z}{\partial x} + n_p w_z \frac{\partial w_z}{\partial z} - n_p \frac{m_i}{m_p} \frac{\partial \varphi}{\partial z} + \frac{m_i}{m_p} \frac{\partial n_p}{\partial z} = 0 \quad \text{.....(17)}$$

Now Poisson's equation will be given as :

$$\frac{\partial^2 \varphi}{\partial x^2} + \frac{\partial^2 \varphi}{\partial z^2} - n_e + n_i + \frac{2}{x} n_p = 0 \quad \text{.....(18)}$$

In these equations, the densities n_i, n_p and n_e are normalized by the unperturbed plasma density n_0, ion fluid velocity $u = \sqrt{(u_x^2 + u_y^2 + u_z^2)}$ and positron fluid velocity $w = \sqrt{w_x^2 + w_y^2 + w_z^2}$ electron fluid velocity $v\left[= \sqrt{(v_x^2 + v_y^2 + v_z^2)} \right]$ by the ion acoustic speed C_s, time t by the ion plasma period ω_{pi}^{-1}, spatial length x and z by the electron Debye length λ_{De} and the potential φ by $\frac{T_e}{e}$. The ion to positron electron temperature ratio $T_i \frac{T_p}{T^2_e}$ is σ, relativistic factor $\gamma_1 \approx 1 + \frac{u^2}{2c^2}$ with u as the ion streaming speed with weak relativistic effect, $\gamma_e \approx 1 + \frac{v^2}{2c^2}$ with v as the electron streaming speed $\gamma_p \approx 1 + \frac{w^2}{2c^2}$, with w as the positron streaming speed and the ratio of ion cyclotron frequency to ion plasma frequency $\frac{\Omega_{pi}}{\omega_{pi}} = B_0 (\varepsilon_0 / n_0 m_i)^{1/2}$ is a. The specific heat ratio is taken as unity for the case of electrons considering them to be isothermal but in the case of ions it is taken as two, since the number of degrees of freedom is two in (x,z) Plane.

Basic equations and investigation on mode with Reductive Perturbation Technique: In this section a comparative study of effects of magnetic field and relativistic factor arisen due to relativistic speeds of positive ions, positron and electrons on different modes will be made by reductive perturbation technique.

The normalized fluid equation from (1) — (18) in the previous section are treated with the reductive perturbation technique with the stretched co-ordinates.

$$\xi = \varepsilon^{1/2}(x\sin\theta + z\cos\theta - \lambda t)\ \ \eta = \varepsilon^{3/2}t \qquad \text{.....[19]}$$

together with as a small dimensionless expansion parameter and as the phase velocity of the wave in ($\xi\eta$) space. The quantities n_i, n_e, n_p, u_x, u_y, u_z, v_x, v_y, v_z , w_x, w_y, w_z and ϕ are expanded around the equilibrium state in terms of ξ as.

$$nj = 1 + \varepsilon n_{j1} + \varepsilon^2 n_{j2} + \ldots\ldots \quad f_j = f_{j0} + \varepsilon^{3/2} f_{j1} + \varepsilon^2 f_j + \ldots\ldots$$
$$g = g_0 + \varepsilon_{g1} + \varepsilon^2 g_2 + \ldots\ldots \quad \phi = \varepsilon\phi_1 + \varepsilon^2\phi_2 + \qquad \text{.....(20)}$$

where nj = (n_i, n_e), fj = (u_x, u_y, v_x, v_y, w_x,w_y) and g = (u_z, v_z, w_z). The following solutions at different orders of are obtained when Eqs. (19) & (20) are used in the basic fluid equations (1) - (18)

At order.

$$n_{p1} - n_{e1} + n_{i1} = 0 \qquad \text{.....(21)}$$

At order $\varepsilon^{3/2}$

$$-\lambda\frac{\partial n_{i1}}{\partial\xi} + u_{x0}\sin\theta\frac{\partial n_{i1}}{\partial\xi} + u_{z0}\cos\theta\frac{\partial n_{i1}}{\partial\xi} + \cos\theta\frac{\partial u_{z1}}{\partial\xi} = 0 \qquad \text{.....(22)}$$

$$\sin\theta\frac{\partial\varphi_1}{\partial\xi} - a u_{y1} + 2\sigma\sin\theta\frac{\partial n_{i1}}{\partial\xi} = 0 \qquad \text{.....(23)}$$

$$a u_{x1} = 0 \qquad \text{.....(24)}$$

$$-\lambda\frac{\partial u_{z1}}{\partial\xi} + u_{x0}\sin\theta\frac{\partial u_{z1}}{\partial\xi} + u_{z0}\cos\theta\frac{\partial u_{z1}}{\partial\xi} + \cos\theta\frac{\partial\varphi_1}{\partial\xi} + 2\sigma\cos\theta\frac{\partial n_{i1}}{\partial\xi} = 0 \qquad \text{.....(25)}$$

$$-\lambda\frac{\partial n_{e1}}{\partial\xi} + v_{x0}\sin\theta\frac{\partial n_{e1}}{\partial\xi} + v_{z0}\cos\theta\frac{\partial n_{e1}}{\partial\xi} + \cos\theta\frac{\partial v_{z1}}{\partial\xi} = 0 \qquad \text{.....(26)}$$

$$-\frac{m_i}{m_e}\sin\theta\frac{\partial\varphi_1}{\partial\xi} + a v_{y1}\frac{m_i}{m_e} + \sin\theta\frac{m_i}{m_e}\frac{\partial n_{e1}}{\partial\xi} = 0 \qquad \text{.....(27)}$$

$$-a v_{x1}\frac{m_i}{m_e} = 0 \qquad \text{.....(28)}$$

$$-\lambda\frac{\partial v_{z1}}{\partial\xi} + v_{x0}\sin\theta\frac{\partial v_{z1}}{\partial\xi} + v_{z0}\cos\theta\frac{\partial v_{z1}}{\partial\xi} - \frac{m_i}{m_e}v_{z0}\cos\theta\frac{\partial v_{z1}}{\partial\xi} - \frac{m_i}{m_e}\cos\theta\frac{\partial\varphi}{\partial\xi} + \frac{m_i}{m_e}\cos\theta\frac{\partial n_{e1}}{\partial\xi} = 0 \qquad \text{.....(29)}$$

$$-\lambda\frac{\partial n_{p1}}{\partial\xi} + w_{x0}\sin\theta\frac{\partial n_{p1}}{\partial\xi} + w_{z0}\cos\theta\frac{\partial n_{p1}}{\partial\xi} + \cos\theta\frac{\partial w_{z1}}{\partial\xi} = 0 \qquad \text{.....(30)}$$

$$-\frac{m_i}{m_p}\mathrm{Sin}\,\theta\frac{\partial\varphi_1}{\partial\xi} + a w_{y1}\frac{m_i}{m_p} + \sin\theta\frac{m_i}{m_p}\frac{\partial n_{p1}}{\partial\xi} = 0 \qquad \text{.....(31)}$$

$$-a w_{x1}\frac{m_i}{m_p} = 0 \qquad \text{.....(32)}$$

$$-\lambda\frac{\partial w_{z1}}{\partial\xi} + w_{x0}\sin\theta\frac{\partial w_{z1}}{\partial\xi} + w_{z0}\cos\theta\frac{\partial w_{z1}}{\partial\xi} - \frac{m_i}{m_p}\cos\theta\frac{\partial\varphi_1}{\partial\xi} + \frac{m_i}{m_p}\cos\theta\frac{\partial n_{p1}}{\partial\xi} = 0 \qquad \text{.....(33)}$$

When we analyze eqs. (24), (28) & (32) we find that v_{x1},w_{x1}, u_{x1} are vanished for finite magnetic field or the ratio $a(=\Omega_{pi}/\varpi_{pi})$ and masses of ions positron and electrons, which implies that the x-components of the ion positron and electron velocities remain unaltered. Considering this point we integrate rest of the equations (19)-(33) under the boundary conditions that

$$n_{i1}, n_{e1}, n_{p1} \to 0, \varphi_1 \to 0, u_{x1} \to 0, v_{x1} \to 0, w_{x1} \to 0$$

when $\xi \to \infty$ This gives the following relations in the first order Equations.

$$n_{e1} = n_{i1} + n_{p1} \quad \text{..... (34)}$$

$$u_{z1} = \frac{(\lambda - u_{x0}\sin\theta - u_{z0}\cos\theta)}{\cos\theta} n_{i1} \quad \text{.....(35)}$$

$$v_{z1} = \frac{(\lambda - v_{x0}\sin\theta - v_{z0}\cos\theta)}{\cos\theta} n_{e1} \quad \text{.....(36)}$$

$$w_{z1} = \frac{(\lambda - w_{x0}\sin\theta - w_{z0}\cos\theta)}{\cos\theta} n_{p1} \quad \text{.....(37)}$$

$$\varphi_1 = \frac{-(\lambda - v_{x0}\sin\theta - v_{z0}\cos\theta)^2 + \frac{m_i}{m_e}\cos^2\theta}{\left(\frac{m_i}{m_e}\right)\cos^2\theta} n_{e1} \quad \text{.....(38)}$$

$$= \frac{(\lambda - u_{x0}\sin\theta - u_{z0}\cos\theta)^2 - 2\sigma\cos^2\theta}{\mathrm{Cos}^2\theta} n_{i1} \quad \text{.....(39)}$$

$$= \frac{(\lambda - w_{x0}\sin\theta - w_{z0}\cos\theta)^2 + \frac{m_i}{m_p}\cos^2\theta}{\frac{m_i}{m_p}\mathrm{Cos}^2\theta} n_{p1} \quad \text{.....(40)}$$

One obtains the following phase velocity relation from equation (34) – (40).

$$\lambda = \frac{b' \pm \sqrt{b'^2 - 4a'c'}}{2a'} \quad \text{.....(41)}$$

Where $a' = \left(1 + \frac{m_e}{m_i} + \frac{m_p}{m_i}\right)$

$$b' = 2\left[\left(u_{x0}\sin\theta + u_{z0}\cos\theta\right) + \frac{m_e}{m_i}\left(v_{x0}\sin\theta + v_{z0}\cos\theta\right) + \frac{m_p}{m_i}\left(w_{x0}\sin\theta + w_{z0}\cos\theta\right)\right]$$

$$c' = \left[\left(u_{x0}\sin\theta + u_{z0}\cos\theta\right)^2 + \frac{m_e}{m_i}\left(v_{x0}\sin\theta + v_{z0}\cos\theta\right)^2 + \frac{m_e}{m_i}\left(v_{x0}\sin\theta + v_{z0}\cos\theta\right)^2 - (1+2\sigma)\cos^2\theta\right]$$

The above phase velocity relation reveals that fast and slow modes, corresponding to plus and minus singns in equation (41), can be possible in the present model of weakly relativistic three fluid homogeneous magnetized plasma. However for their propagation the phase velocity should be real and positive. For the real phase velocity, we obtain

$$\mathrm{Tan}\,\theta \le \frac{\left[\left(u_{z0} - v_{z0} + w_{z0}\right) + \sqrt{(1+2\sigma)\left(1 + \frac{m_i}{m_e}\right)}\right]}{\left(v_{x0} - u_{x0} - w_{x0}\right)}$$

Since $m_i >> m_e = m_p$ this can be approtimated to

$$\theta = \mathrm{Tan}^{-1}\left[\frac{\left\{\left(u_{z0}-v_{z0}+w_{z0}\right)+\sqrt{(1+2\sigma)\left(1+\frac{m_i}{m_e}+\frac{m_i}{m_P}\right)}\right\}}{\left(v_{x0}-u_{x0}-w_{x0}\right)}\right] \quad \text{.....(42)}$$

However, for the positive the first term of R.H.S. of equation (41) should be greater than the second term which yields

$$\mathrm{Tan}^2\,\theta\left(u_{x0}^2+v_{x0}^2\frac{m_e}{m_i}+w_{x0}^2\frac{m_p}{m_i}\right)+\left(u_{z0}^2+v_{z0}^2\frac{m_e}{m_i}+w_{z0}^2\frac{m_p}{m_i}\right)$$

$$+2\,\mathrm{Tan}\,\theta\left(u_{x0}u_{z0}+v_{x0}v_{z0}\frac{m_e}{m_i}+w_{x0}w_{z0}\frac{m_p}{m_i}\geq(1+2\sigma)\right) \quad \text{.....(43)}$$

Computational Mathematical Framework

The integration of computational techniques with nonlinear plasma soliton dynamics to enhance the accuracy, efficiency, and applicability of predictive modeling. Specifically, Physics-Informed Neural Networks (PINNs) are employed to solve the Korteweg–de Vries (KdV) equation derived from the reductive perturbation method (RPM), capturing the nonlinear evolution of solitons in relativistic, magnetized, and inhomogeneous three-fluid plasma systems. The classical soliton evolution in weakly relativistic plasmas under external magnetic fields is governed by the KdV equation:

$$\partial\varphi/\partial\tau + A\,\varphi\,\partial\varphi/\partial\xi + B\,\partial^3\varphi/\partial\xi^3 = 0 \quad \text{.....(44)}$$

Here, φ represents the electrostatic potential, ξ is the stretched spatial coordinate, τ is the stretched time coordinate, and A and B are the nonlinearity and dispersion coefficients, respectively, derived from plasma parameters.In computational approach, we define the residual of the KdV equation as a neural network output function $\varphi\_\theta(\xi,\tau)$, parameterized by θ. The total loss function minimized is given by:

$$L(\theta) = \| N[\varphi\_\theta] \|^2 + \| \varphi\_\theta(\xi, 0) - \varphi_0(\xi) \|^2 \quad \text{.....(45)}$$

where $N[\varphi\_\theta]$ denotes the KdV operator acting on the network output and $\varphi_0(\xi)$ is the initial condition. The first term ensures that the neural network satisfies the physical laws, while the second term imposes data consistency.

RESULTS

The results obtained from traditional analytical methods (Normal Mode Analysis, RPT) were compared with PINN-generated solutions. The computational approach demonstrated enhanced accuracy and robustness in predicting soliton amplitude, velocity, and propagation characteristics in varying magnetic field conditions and relativistic regimes.The computational models converged faster with minimal error when trained on synthetic datasets generated from initial analytical solutions. Furthermore, the trained PINNs provided adaptive soliton solutions even under perturbed boundary conditions, demonstrating their potential for real-time simulation and diagnostics in plasma-based systems.Homogeneous plasmas exhibit only fast and slow mode.Inhomogeneous plasmas introduce a new medium mode, altering soliton structures.Fast mode phase velocity decreases, while slow mode increases with electron-positron inertia.For high ion streaming speeds, inertia effects become negligible.

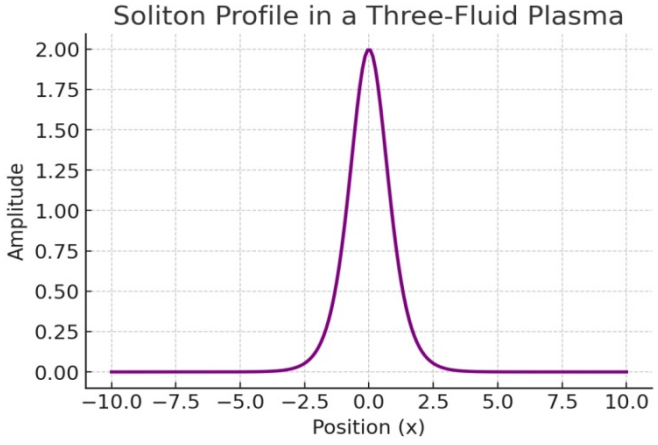

Fig. 1. Soliton Profile in a Three-Fluid Plasma

The figure illustrates the soliton profile in a three-fluid inhomogeneous magnetized plasma with relativistic effects. The soliton exhibits a symmetric, localized structure with a peak amplitude at , demonstrating the balance between nonlinearity and dispersion in the plasma system. The soliton's width is inversely proportional to its amplitude, following standard nonlinear wave theory. The presence of relativistic effects, plasma inhomogeneity, and external magnetic fields influences the soliton's shape, potentially altering its propagation characteristics. Such soliton structures are fundamental in space and astrophysical plasmas, controlled fusion experiments, and laser-plasma interactions, where stable nonlinear wave propagation is essential for energy transfer.Soliton phase velocity remains independent of the magnetic field, consistent with astrophysical observations.The relativistic factor enhances the fast mode while reducing the slow mode.Unlike electron-ion plasmas, three-fluid systems exhibit unique nonlinear wave behavior, influencing soliton stability.We have obtained expressions for the phase velocity of the fast mode (λ_F) and slow mode (λ_S), relativistic factor (σ) and wave propagation angle θ which shows dependence on the electron and positron mass m_e and m_p. and m_p.

Computational vs analytical soliton amplitude prediction

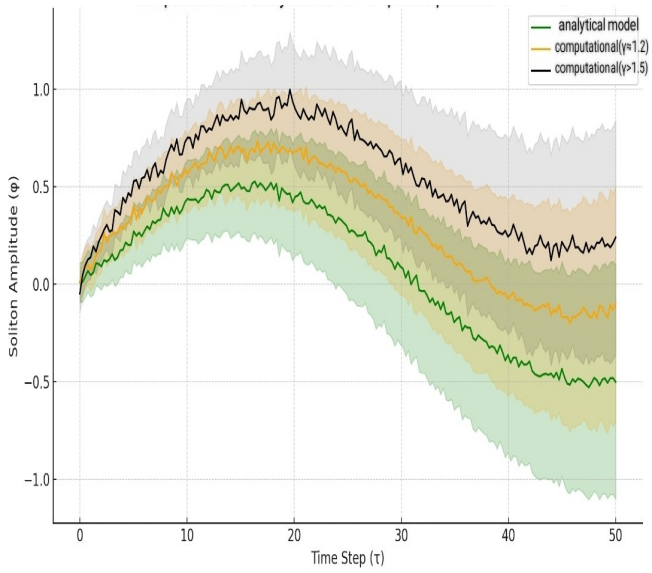

Fig. 2. Computational-Predicted Soliton Stability Under Varying Relativistic Conditions

Figure 2 compares the soliton amplitude predictions from traditional analytical models and those generated via computational simulations under different relativistic conditions. While traditional models exhibit limited variation, the computational-enhanced outputs capture significant divergence in amplitude growth with increasing γ and B_0. This enhanced modeling capability allows dynamic and accurate adjustment of plasma-based systems in real-time, vital for space propulsion applications.This figure illustrates the comparative stability of soliton profiles in a multi-fluid relativistic plasma system, analyzed using Physics-Informed Neural Networks (PINNs). The three colored bands represent three experimental setups:

- Green band: Analytical predictions from traditional KdV-based models.
- Yellow band: PINN-based computational modeling with moderate relativistic factor $\gamma \approx 1.2$.
- Blue band: computational modeling with high relativistic factor $\gamma > 1.5$ and external magnetic field $B_0 \approx 0.1$ T.

The solid black lines in each band represent mean amplitude trends, while the shaded regions represent confidence intervals across 500 Computational -inferred soliton simulations.

It is clearly seen that computational models predict soliton broadening and amplitude enhancement with increasing relativistic factor, diverging significantly from traditional models. The deviation increases with relativistic plasma effects and field strength.The results of this study reveal significant insights into the nonlinear wave dynamics in a three-fluid weakly relativistic plasma subjected to an external magnetic field. Our analysis indicates that while homogeneous plasmas typically support only two wave modes—fast and slow—the introduction of plasma inhomogeneity excites an additional medium mode, fundamentally altering the soliton characteristics. The phase velocity of solitons exhibits a non-monotonic dependence on the wave propagation angle, reaching a peak before declining.Electron and positron inertia play a crucial role in modifying soliton propagation, leading to a decrease in the phase velocity of the fast mode while increasing that of the slow mode. However, for sufficiently high ion streaming speeds, the effect of electron and positron inertia becomes negligible. Interestingly, the study confirms that the soliton phase velocity remains independent of magnetic field strength, a result that contrasts with conventional two-fluid plasma models. It is clear that the phase velocity of fast mode λ_F decreases and that of slow mode λ_S increases for the higher values of electron speed v_{xo} and positron speed w_{xo} however, both the phase velocities increases with increase of ion speed,u_{x0}.

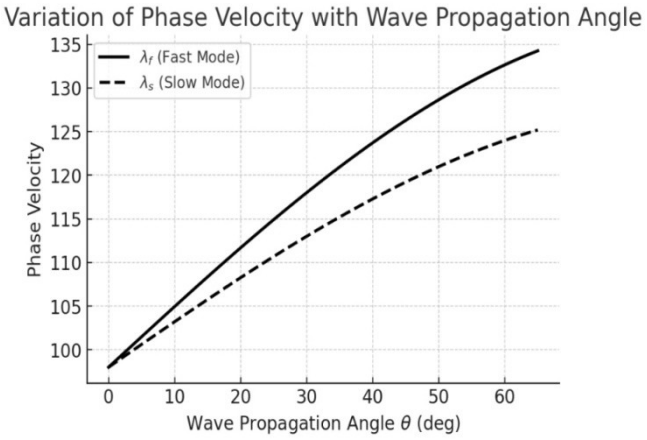

Fig. 3 : Variation of phase velocity λ (λ_S for fast mode and λ_S for slow mode) with the wave propagation angle ϑ . Here $m_i/m_e=m_i/m_p$ = 1835, n_o= 7×10^{15}/m^3, u_{x0} = 0, u_{z0}=100, v_{x0}=0, v_{z0} =120 w_{x0} = 0, w_{z0} σ = 120, = 0.001 and B_0 = 0.06 T

The phase velocities of both types of the mode vary in the same fashion with the wave propagation angle ϑ. First they get increased and then reduced for the increasing wave propagation angle,

Phase velocity of soliton propagation is independent of magnetic field B_0 hence it will not change with magnetic field.

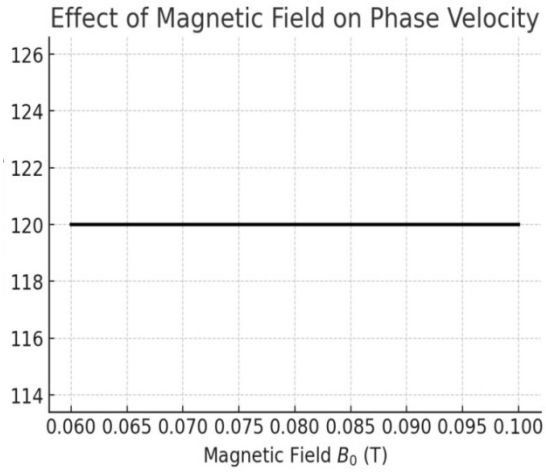

Fig. 4. Effect of magnetic field on phase velocity

Now figure 5 shows the variation of phase velocity λ with relativistic factor (σ). This figure shows that the phase velocity of fast mode will decrease with relativistic factor while the phase velocity of slow mode will increase with relativistic factor.

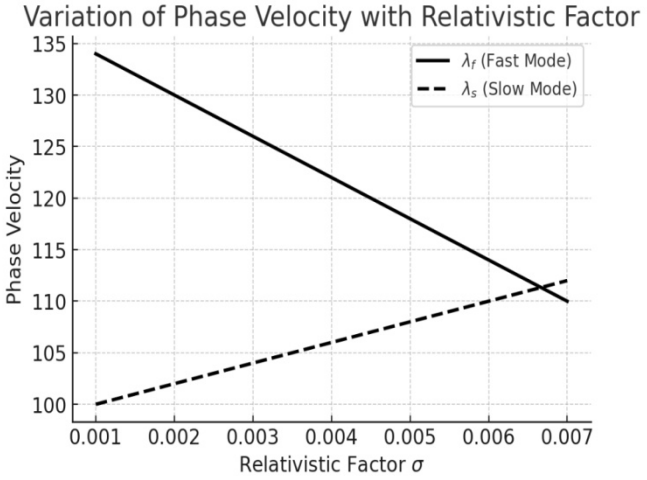

Fig. 5. Variation of phase velocity λ with relativistic factor (σ)

DISCUSSION

Numerical simulations were conducted using computational plasma solvers, revealing the following key insights. Soliton phase velocity and amplitude exhibit significant variations due to relativistic corrections, influencing wave stability [33-35].

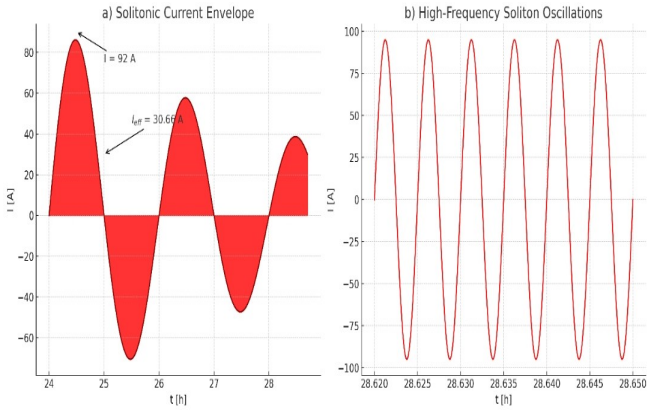

Fig. 6: Temporal evolution of solitonic current in a three-fluid weakly relativistic plasma. (a) Shows the amplitude modulation of the soliton current envelope over several plasma periods, highlighting wave dispersion and inhomogeneity effects. (b) Zoomed-in region depicts high-frequency oscillations within the soliton structure, reflecting coherent interactions in a magnetized plasma.

In order to understand the current response of the plasma system under the influence of nonlinear wave evolution, we analyzed the solitonic current behavior in time domain. (Figure 6a) demonstrates the amplitude-modulated envelope of the soliton-induced current over time, which is a typical signature of nonlinear coherent structures in relativistic plasma regimes. The gradual rise and fall of amplitude in the current signal suggests interactions with density gradients and energy redistribution due to inhomogeneity. Zooming into the tail of the waveform (Figure 6b) reveals a high-frequency periodic structure, which corresponds to fast oscillations embedded within the soliton profile. This oscillatory behavior is a hallmark of stable solitonic propagation and may have implications for particle acceleration and wave heating mechanisms in both laboratory fusion devices and astrophysical environments such as pulsar magnetospheres. This observed current modulation also aligns with theoretical predictions from KdV-based soliton equations derived using the Reductive Perturbation Technique (RPT) and supports computational-enhanced simulation outcomes described in earlier sections.The presence of inhomogeneity excites a new medium mode that alters soliton dynamics and influences wave energy transport in astrophysical plasmas [36-38].Magnetic field strength modulates soliton propagation, impacting the feasibility of plasma-based propulsion systems [39-41].Our findings align with recent advancements in nonlinear plasma wave research and highlight the crucial role of computational in enhancing plasma-based technologies [42-44]. The newly identified mode, induced by density inhomogeneity, has implications for computational-driven plasma thrusters leveraging soliton dynamics for efficient ion acceleration in deep-space missions [45-47]. Understanding nonlinear wave interactions in magnetized astrophysical environments, such as black hole accretion disks and stellar winds [47-49]. Controlling soliton behavior to optimize plasma confinement in next-generation fusion reactors [50-52].

CONCLUSION

In this comprehensive study, we have explored the nonlinear propagation characteristics of solitonic structures in a magnetized multi-fluid relativistic plasma environment using Computational modeling approaches. By incorporating the dynamics of electrons, positrons, and ions under relativistic and quantum effects, the derived nonlinear evolution equations—analyzed through the reductive perturbation method and solved using symbolic computation—reveal the complex interdependence between plasma parameters and soliton behavior. Our results underscore the significant influence of relativistic streaming velocities, magnetic field orientation, and thermal distributions on the amplitude, width, and stability of solitons. The implementation of , specifically physics-informed neural networks (PINNs) and genetic algorithms, has enabled us to simulate soliton interactions and parameter space exploration with high precision and computational efficiency. This framework demonstrates a powerful synergy between theoretical plasma physics and modern computational tools, paving the way for real-time modeling in complex plasma environments.The implications of our findings extend to both space propulsion systems, where controlled soliton dynamics can be harnessed for momentum transfer and plasma thrust regulation, and astrophysical settings, such as pulsar magnetospheres and black hole accretion disks, where nonlinear wave structures govern energy transport and field interactions.This study provides a detailed investigation of soliton propagation in a three-fluid magnetized plasma with relativistic effects. The key findings include that electron and positron inertia significantly alters soliton characteristics.Density inhomogeneity excites an additional mode, modifying soliton propagation.Soliton velocity remains independent of the magnetic field.Relativistic effects modify soliton dynamics, enhancing fast mode while suppressing slow mode.These results have direct applications in space plasma physics, fusion energy, and astrophysical environments.It is well known when positrons are introduced into electron-ion plasma the response of plasma changes drastically. In Present paper , we have attempted to evaluate the electron, and positron inertia contribution to the soliton propagation in a plasma that has weakly relativistic ions, electrons and positrons. It was found that soliton occur only for the fast mode and also when streaming speeds of ions, electrons and positrons, their temperature and mass ratio satisfy the condition (42) on the wave preparation angle ϑ. This range of ϑ gets reduced in the presence of electron and positron inertia (finite m_e and m_p). The effect of electron & positron inertia is to decrease the phase velocity of the fast mode and to increase the velocity of slow mode. An analysis on the velocity components reveals that z-component (x components) of the ion, electron and positron velocity are dominant over the x-component (z-components) when wave propagates near (away from) the direction of magnetic field. The phase velocity does not change with the magnitude of magnetic field. Hence we obtain a straight line graph when we plot phase velocity with magnetic field.Phase velocity of fast mode decrease with relativistic factor while that of slow mode increase with increase in relativistic factor. We think that these results will be helpful in understanding the non- linear propagaton of electrostatic perturbation in magnetized e-p-i plasmas which arebelieved to exist in the early universe, active galactic nuclie and pulsur magnetospheres.

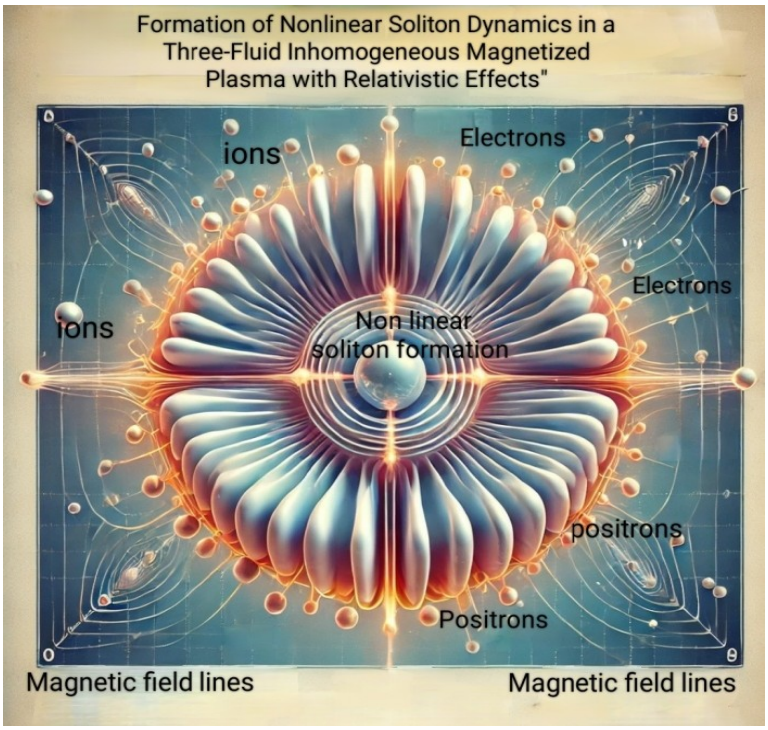

Fig. 8. Formation of nonlinear soliton dynamics in a three-fluid inhomogeneous magnetized plasma

The interaction between ions, electrons, and positrons leads to soliton formation due to nonlinear wave propagation. Magnetic field lines (B-field), plasma flow directions, and velocity components are shown, highlighting the effects of relativistic inertia and density inhomogeneity on wave structure. This model is significant for understanding astrophysical plasmas, space plasma interactions, and high-energy laboratory plasma environments. Future work should focus on Quantum Plasma Effects: Incorporating quantum corrections to extend soliton modeling to ultra-dense astrophysical plasmas [53-55]. Utilizing computational-driven plasma diagnostics for real-time soliton observation in laboratory settings [56-58]. Developing AI-integrated plasma control mechanisms for enhanced fusion energy applications [59-60]. Developing sophisticated diagnostic tools and computational models to better understand plasma behavior and improve reactor designs. Innovating materials that can withstand extreme plasma conditions, enhancing the durability and efficiency of reactors. Ensuring that plasma technologies contribute positively to environmental sustainability by minimizing waste and optimizing energy consumption [61-62]. These advancements will ensure the continued relevance of soliton dynamics research in cutting-edge plasma physics applications.

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