ISSN: 2230-9926

Available online at http://www.journalijdr.com

IJDR

International Journal of Development Research
Vol. 15, Issue, 02, pp. 67658-67663, February, 2025
https://doi.org/10.37118/ijdr.29079.02.2025

RESEARCH ARTICLE OPEN ACCESS

ESTIMATING A GARCH MODEL FOR GOLD PRICE RETURNS: A BAYESIAN APPROACH

Sivasamy, R.*[1,2], Thupeng, W.M.[2], Adebayo, F.[2] and Manyeagae, G.D.[2]

[1]Annamalai University –Retired, Annamainagar 608002, Chidambaram, Cuddalore District, India
[2]University of Botswana, Department of Statistics, Gaborone, Botswana

ARTICLE INFO

Article History:
Received 02nd December, 2024
Received in revised form
11th December, 2024
Accepted 19th January, 2025
Published online 27th February, 2025

Key Words:
Return, Volatility, GARCH, Student-t, MCMC, Estimates.

Corresponding author: Sivasamy, R.,

ABSTRACT

The frequency of rerun of any stock consistently fluctuates due to the competing forces of supply and demand responding to changes in the share prices by investors. Historically, gold prices have generally produced favourable returns during both challenging and thriving times, positioning gold as a means for safeguarding and enhancing wealth. In this study, we present a Bayesian Generalized Auto-Regressive Conditional Heteroskedastic (GARCH) volatility model for daily gold price returns based on the most recent 2500 daily prices. The present research aims to showcase the use of the stan-garch function from the 'bayesforecast' of (R-package) to fit a GARCH (1, 1) model to the gold price returns data, assuming Student-t and normal error distributions. The results of the research show that the model effectively captures the data. It is also concluded that the effects of prior shocks will result in a lasting impact on the future volatility of the daily gold price returns.

Citation*: *Sivasamy, R., Thupeng, W.M., Adebayo, F. and Manyeagae, G.D. 2025. "Estimating a garch Model for gold Price returns: A Bayesian Approach*". International Journal of Development Research*, 15, (02), 67658-67663.

INTRODUCTION

Bayesian statistical conclusions about a parameter θ, are made in terms of probability statements using an observed sample value, say y, collected from the concerned population. These probability statements, p(θ/y) are conditional on the observed value of y. Let the unconditional distribution of θ be p(θ), called 'prior" and the joint distribution of y and θ be p(y,θ), called likelihood, Then, the Bayes rule links, p(θ) and p(y,θ), to get the conditional posterior distribution

$$p(\theta/y) = \frac{p(\theta)\ p(y,\theta)}{\int \ p(\theta)\, p(y,\theta) d\theta}$$

Markov chain simulation (also called Markov chain Monte Carlo or MCMC) is a general technique based on drawing θ values from approximate distributions and adjusting them to better approximate posterior distribution, *p*(θ/*y*). The sampling is to be done sequentially, with the distribution of the sampled drawings depending on the last value drawn; hence, the drawings form a Markov chain (MC). The transition probability distributions of the MC must be constructed so that the Markov chain converges to a unique stationary distribution that is the posterior distribution, *p*(θ/*y*). Several books provide sampling algorithms for implementing MC simulation methods: the Gibbs sampler and the Metropolis-Hasting algorithm are computational approaches based on approximation.

Since the R language is an important tool for time-series analysis, in this paper we use the Rpackages Bayesforecast and MSGARCH to model and estimate the value of GARCH and density forecasts.

ARCH and GARCH Models: An autoregressive conditionally heteroskedastic (ARCH) model is a statistical model used to analyse historical volatility to predict future volatility. The GARCH model is an extension of the ARCH model that also allows for variance in the error term. ARCH models were created in the context of econometric and finance problems having to do with the amount x(t) of investments in stock that may increase (or decrease) per period. Our interest in this case is to formulate the best model on

$$y_t = \log(x_t) - \log(x_{t-1}) \qquad \text{………………..(1)}$$

y_t of (1) is the logarithm of the ratio of this time's value to last time's value.

GARCH models are used when the variance of the error space is unequal over time. That is, the error term is heteroskedastic. Heterogeneity describes the irregular variation pattern of an error term or variable in a statistical model. Essentially, wherever there is heteroskedasticity, observations do not conform to a linear pattern, but they tend to cluster. In particular, the variance of the error term in GARCH models is assumed to vary systematically, conditional on the average size of the error terms in previous periods.

LITERATURE REVIEW

Many details on the Bayesian Estimation of GARCH Models can be found in (Ardia(2008a)), (Ardia(2008b)), (Ardia(2009)), (Ardia and Hoogerheide,(2010)), (Ardia, Keven, Kris, Leopoldo, Denis-Alexandre (2019)), and (Bollerslev (1986)). A few asymmetric GARCH-type models for asymmetric volatility characteristics analysis and regim switching have been discussed in (Chen *et al*., (2019)), (Maciel, Leandro (2021)). Non-parametric methods and volatilityvolatility estimation methods for financial risk factors have been discussed in (Naik and Mo- Han (2021)),(Makatjane and Moroke (2022)) and (Oseifuah and Korkpoe (2019)). A dynamic volatility modelling of Bitcoin using a time-varying transition probability Markov-switching GARCH model is well studied by (Tan et al. (2021)). (Vats and Knudson (2020)) revisited the Gelman-Rubin Diagnostic, and thus made some improvements towards the estimation of GARCH models. (Xiao, Yang (2021)) introduced a forecasting extreme risk using regime-switching GARCH models. (Villa and Walker (2014)) used objective prior type for the number of degrees of freedom of a t distribution in their Bayesian analysis of time series.

Organization of Sections: Section 2 deals with GARH's model for retrieving daily price charts for gold stocks. Section 3 describes the prior and posterior distributions of the parameters of the studied GARCH (1,1) model. Section 4 obtains additional results on the estimation of GJRGARCH with the 'MSGARCH' package. Section 5 presents the conclusion.

GARCH Model for Gold Price Returns: Understanding volatility is straightforward, yet its modelling proves to be challenging. In the realm of finance, volatility refers to standard deviation, highlighting the extent to which the values of financial assets fluctuate. Volatility modelling is grounded in the rate of return of an asset, also known as realized volatility, to grasp uncertainty and provide us with reliable estimates of reality. How are we prepared to enhance predictive performance by leveraging both the Bayes framework model and machine learning models such as support vector regression, neural networks, and deep learning? This will enable us to effectively compare the predictive capabilities.

The main aim of the ARCH model introduced by (Engle (1982)) is to describe the variance σ^2 of a random variable with the following equation:

$$\sigma_t^2 = \nu V_t + \sum_{i=1}^{q} \alpha_i r_{t-i}$$

- r_t is the return series of the stock price whose variance σ^2 is to be modeled.
- V_L is the long-term variance of stock.
- γ and α are weights that satisfy $\gamma + \Sigma\, \alpha\_i = 1$
- q is the order of the auto-regressive process, i.e., is ARCH(q).
- $\gamma\, V_L$ is often expressed as ω or α_0

GARCH (1, 1) Model

A GARCH (1,1) is

$$Var(y_t \mid y_{t-1}) = \sigma_t^2 = \alpha_0 + \alpha_1\, y_{t-1}^2 + \beta_1\, \sigma_{t-1}^2 \quad (2)$$

where, $\alpha_0 \geq 0$, $\alpha_1 \geq 0$ and $\beta_1 \geq 0$ to avoid negative variance. Define the term y_t as a function of a white noise $\epsilon_t \sim N(0,1)$ and inverted gamma function IG i.e., λ_t as in

$$y_t = \mu + \varepsilon_t \sqrt{\left(\frac{\gamma-2}{\gamma}\lambda_t\, \sigma_t\right)} \quad ; t \geq 1 \quad (3)$$

where λ_t follows inverted gamma IG ($\nu/2$, $\nu/2$) function, $\nu > 2$ is an integer. The restriction on the degrees of freedom parameter $\nu > 2$ guarantees the finite conditional variance, and the restrictions on the GARCH parameters $\alpha_0 > 0$, $\alpha_1 > 0$, and $\beta_1 > 0$ ensures positive variance σ^2.

We utilize Bayesian estimation techniques on daily observations of gold price log returns, focusing on 2500 data points. This allows us to generate forecasts for n = 252 values at the tail end. To identify any unusual observations, we first plot the y_t series in Figure 1 and the corresponding forecast version in Figure 2.

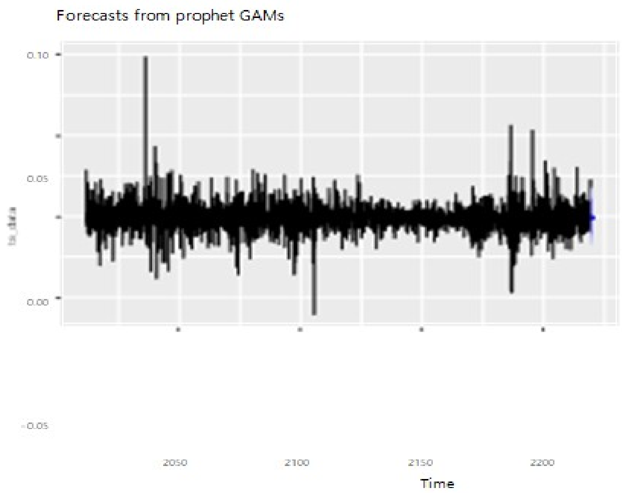

Figure 1. Plot for original y_t to identify any unusual observations

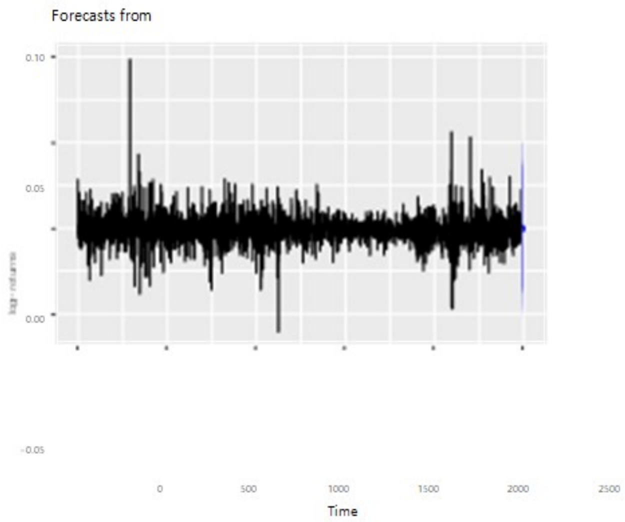

Figure 2. Plot based on the predicted y_t for the next few days

Prior and Posterior distributions of all parameters of GARCH (1,1) Model: Several R packages provide functions to estimate GARCH models using the maximum likelihood method such as 'fGarch' and 'rgarch'. The bayesian theory offers a valuable substitute for modeling outcomes, assessments, model variability, model groups, and probabilistic assertions regarding functions (which can be nonlinear) of model parameters. In the model proposed we use truncated normal priors on the GARCH parameters $\boldsymbol{\beta}$ and $\alpha = (\alpha_0, \alpha_1)'$. The prior distributions p(α) on $\alpha = (\alpha_0, \alpha_1)$ is a bivariate truncated Normal distribution:

$p(\alpha)$ a $N_2(\mu_\alpha, \Sigma_\alpha)\, I_{(\alpha>0)}$ where $I_{(\alpha>0)}$ is the indicator function (4)

The prior distribution p(β) on β is a univariate truncated Normal distribution:

$p(\beta)$ a $N(\mu_\beta, \Sigma_\beta)\, I_{(\beta>0)}$ where $I_{(\beta>0)}$ is the indicator function (5)

Table 1. Creation of the GARCH (1,1) Model through the *stan-garch* function

	mean	SE	5%	95%	ESS	$\hat{R}$
μ_0	-0.0002	0.0000	-0.0004	0.0001	933.1439	0.9999
α_0	0.0009	0.0000	0.0004	0.0016	934.6304	1.0052
arch	0.6598	0.0067	0.2680	0.9648	967.8143	1.0007
garch	0.4574	0.0083	0.0557	0.9026	1065.6318	0.9992
v	2.0362	0.0005	2.0159	2.0628	976.0091	1.0011
loglik	8612.7616	0.7053	8577.1698	8649.3006	924.0692	1.0012

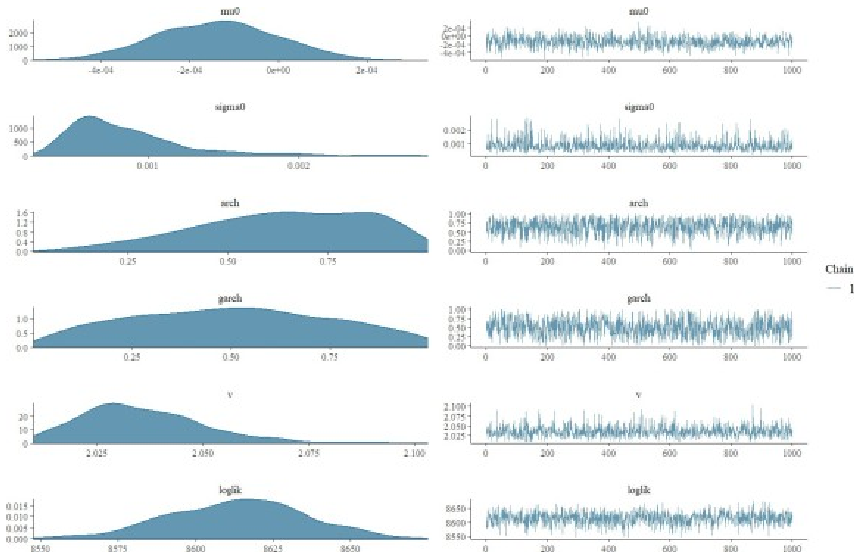

Figure 3. Plot for the estimated posteriors

The prior distribution of vector $\lambda = (\lambda_1, \ldots, \lambda_T)'$ conditional on v is found by noting that the components λ_t are independent and identically distributed from the inverted gamma, which yields posterior information.

The prior distribution p (λ/ v) is a truncated exponential distribution:

$$p(\lambda|v) = \left(\frac{v}{2}\right)^{\frac{Tv}{2}} \left[\Gamma\left(\frac{v}{2}\right)\right]^{-T} \left(\prod_{t=1}^{T} \lambda_t\right)^{-\left(\frac{v}{2}+1\right)} exp\left[-\frac{1}{2}\sum_{t-1}^{T} \frac{v}{\lambda_t}\right] \quad (6)$$

The prior distribution on the degrees of freedom parameters is a translated exponential with parameters $\lambda^*>0$, $\delta \geq 2$

$$p(v) = \lambda^* \, exp\,[-\lambda^* (v - \delta)] I_{(v>\delta)} \quad (7)$$

For high values of λ, the prior mass is focused around δ, and the limited degrees of freedom can be established. All of these were specified by Stan's suggested guidelines. The posterior distributions are illustrated in Figures 3 and 4. It is noted that the posteriors are converging. Moreover, while adhering to the model's fit, it does not fully accommodate the data as the model fails to represent the dependency structure in the location parameter. Next, we move on to examine the model residuals. The check residual's function computes the posterior mean of the residuals and generates a plot. Keep in mind that the plot alone is inadequate to confirm the normality and stationarity assumptions, but they serve as a preliminary indication of the adjustment. The residual series (Upper part) of Figure 3 appears to be stationary. Nevertheless, the histogram and quantile graph (middle part) indicate that the model exhibits heavy tails because of the series' significant volatility. Drawing from these or earlier findings, we ultimately forecast the model for the upcoming days in Figure 4.

Markov-switching GARCH model for $\{y_t\}$: GARCH models within Markov switching frameworks have gained popularity as techniques to capture volatility alterations in a time series' conditional variables. The R package MSGARCH enables us to carry out simulations, maximum likelihood, and Bayesian Markov chain Monte Carlo estimators for an extensive range of GARCH models. The package offers methods for forecasting one-step and multi-step for the entire conditional density of the variable we are focused on. Tools for risk management are also offered to assess conditional volatility, value at risk (VoR), and expected shortfall. The paper introducing the original autoregressive conditional heteroscedasticity (ARCH) model is (by Engle (1982)), while its generalization to GARCH was introduced by (Bollerslev (1986)) and (Engle and Ng(1993)). Also, (Ardia (2009)) discussed the Bayesian estimation of a Markov- Switching Threshold Asymmetric GARCH Model with Student-T Innovations. To obtain true changes in volatility relative to regime changes, the parameters of the GARCH model must change over time in a hidden Markov fashion. This approach is called the Markov Switching GARCH (MSGARCH) model, which produces value forecasts that can adapt to changes in the zero-volatility level. Assume that the expected value of log return series y_t is zero and $\{y_t\}$ is serially uncorrelated. Following (Ardia *et al,* (2018)), we formulate our GARCH model as below:

$$y_t/(s_t = k, I_{(t-1)}) \sim D(0, h_{k,t}, \xi_k), \;\; h_{k,t} = h(y_{(t-1)}, h_{(t-1)}, \theta_k) \quad (8)$$

where $D(0, h_{k,t}, \xi)$ is a continuous distribution with a zero mean, a time-varying conditional variance $h_{k,t}$ in regime k, and a vector ξ_k. of additional shape (e.g., tail and asymmetry) parameters and the additional regime-dependent vector of parameters is θ_k. The state variable s_t evolves according to a first–order homogeneous Markov chain with a finite number of states. K. In addition, $I_{(t-1)}$the term of the equation (8) denotes the information set available up to $(t-1)$ and θ_k denotes the regime–dependent vector of parameters. Following (Bollerslev (1986)), we have a symmetric GARCH(1,1) model as in the statement (9):

$$h(k,t) = \alpha 0,k + \alpha k\, y^2\, 1 + \beta k \;\; h(k,t-1) \quad (9)$$

Following (Glosten, Jagannathan, and Runkle (1993)), we suggest an asymmetric GJRGARCH (1, 1) model as in the equation (10):

$$h(k,t) = \alpha 0,k + (\alpha 1,k + \alpha 2,k\, I\{y(t-1) < 0\})y^2\, 1 + \beta k\, h(k,t-1) \quad (10)$$

where $I\{y_{(t-1)} < 0\}$ is an indicator function.

State Process s_t : We assume that the underlying state process s_t evolves on a state space $S = \{1, 2, \ldots, K\}$ according to an unobserved first-order ergodic homogeneous Markov chain with transition probability matrix P= ($p_{i,j}$= P(s =j/s_{t-1} = i)) is the probability of a transition from state s_{t-1} to s_t.

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1k} \\ p_{21} & p_{22} & \cdots & p_{2k} \\ \vdots & \vdots & \cdots & \vdots \\ p_{k1} & p_{k2} & \cdots & p_{kk} \end{pmatrix} \quad (11)$$

We estimate the model parameters with a Bayesian approach (via MCMC simulation). This allows us to write interesting probability expressions on (non-linear functions of) model parameters, such as the leverage effect and the free variance in each mode k. We use diffuse priorities and simulate Metropolis adaptive random samples to generate draws from the posterior. In addition, we impose restrictions on the parameters to ensure that the function of those volatilities under the MSGARCH specification cannot be generated by a single–regime specification. Both are achieved through the prior specification.

Table 2 shows the estimates of the parameters of the GJRGARCH Model through ML and MCMC methods with several regimes k=2. Under each of the ML and MCMC approaches, the stationary distribution $\{\pi_1, \pi_2\}$ of switching between the two regions is also computed to study the long-run effects.

In-sample analysis of the daily log returns of gold prices: We now centre on an empirical outline utilizing the later 4151 day's log returns of our gold price information set. We outline how the R bundle MSGARCH can be utilized to demonstrate comparison, state/regime smoothing, and instability shifting. Estimation through MCMC is additionally examined. The plot of the time series containing 2500 data points is presented in Figure 1. Well-known stylized truths observed in financial time series, such as volatility clustering and nearness of outliers, are apparent in Figure 1. For the sake of illustration, we consider the asymmetric two-state MSGARCH model, where a GJRGARCH variance specification with a student-t distribution is assumed in each regime. We then fit the model with the ML estimation to the same data set and the results are

Table 2. Estimation of the GJRGARCH Model through ML and MCMC methods with k=2

parameter	ML estimate	Method se	MCMC mean	posterior mean sd
$\alpha0,1$	0	0	0	0
$\alpha1,1$	0.0224	0.005	0.0074	0.0227
$\alpha2,1$	0.0002	0.0006	0.2005	0.1262
β_1	0.977	0.005	0.259	0.1694
v_1	4.8087	0.3558	5.4915	0.5554
$\alpha0,2$	0	0	0	0
$\alpha1,2$	0	0.0002	0.0547	0.0075
$\alpha2,2$	0.1279	0.0273	0.0007	0.0065
β_2	0.9318	0.0143	0.9405	0.0078
P1,1	0.978	0.0154	0.046	0.0485
P2,1	0.0431	0.0278	0.0559	0.0475
stationary	π_1=0.662	π_2=0.338	π_1=0.0554	π_2=0.9446

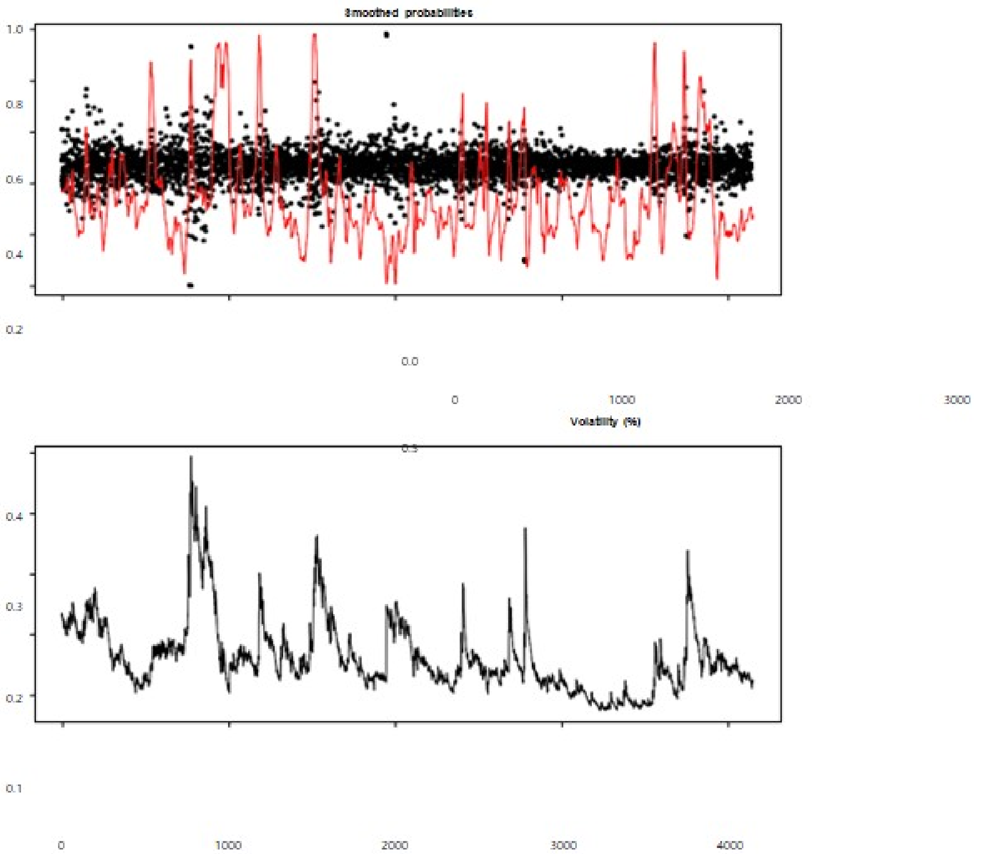

Figure 5. Top graph: The smoothed probabilities for regime two. Bottom graph: The filtered volatility of the overall process

ordered concerning the unconditional variance in each regime, from lower to higher values, when regimes have the same model specification. The outputs of fit.ml = FitML (spec = ms2.garch.n, data= gold price returns) and that of fit.mcmc =FitMCMC (spec = ms2.garch.n, data = gold price returns) are given in Table 2. Parameter estimates of Table 2 show that the evolution of volatility is heterogeneous across the two regimes. Without a doubt, we first note that the two regimes report different unconditional volatility levels: a different reaction to past returns: $\alpha_{2,1}$ = 0.0002 vs. $\alpha_{2,2}$ = 0.1279. The first regime reports $\alpha_{1,1}$+(1/2) $\alpha_{2,1}$+β_1 =0.9995 and the second regime has $\alpha_{1,2}$+ (0.5) $\alpha_{2,2}$ +β_2 =0.99575 which implies that the first regime has higher persistence of the volatility process than that of the second regime. To evaluate the goodness–of–fit of the models, we use the Deviance information criterion (DIC) obtained from the MCMC estimation. Other useful statistics are the acceptance rate which, for the MCMC sampler, is 28,5 % and the deviance information criterion (DIC) of the test is -27267.560. Filtered, anticipated, and smoothed probabilities can be computed beginning from evaluated objects utilizing the State (.) function. It produces Viterbi records as a matrix of measurement, speaking to decoded states agreeing to the Viterbi algorithm point by point in (Viterbi (1967). For instance, Smooth probabilities of being within the second regime (k = 2) can be computed with the following code: *smooth.prob = State (fit.ml) Smooth Prob*[*,* 1*,* 2*, drop* = *TRUE*]

Figure 6 displays the smoothed probabilities of being in regime two (high unconditional volatility regime), superimposed on the gold price log-returns (top graph) as well as the filtered volatility of the overall process (bottom graph). Volatilities are extracted from the estimated object using the function Volatility. As expected, when the smoothed probabilities of regime two are near one, the filtered volatility of the process sharply increases. Interestingly, we further note that the Markov chain evolves persistently over time and that, in the limit, as reported by the ML method (Stable probabilities), the probabilities of being in the two states are about 5.54% and 94.46%.

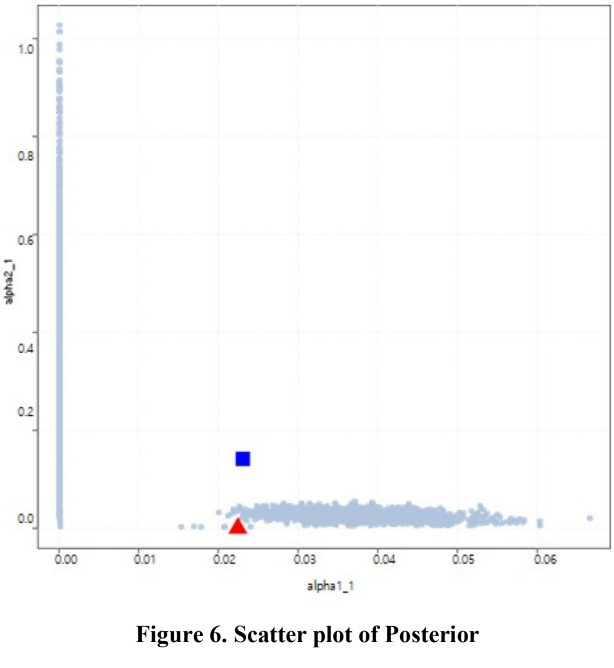

Figure 6. Scatter plot of Posterior

Figure 6. Scatter plot of posterior draws from the marginal distribution of $\alpha_{1,1}$ and $\alpha_{1,2}$ obtained with the adaptive random walk strategy. The blue square reports the posterior meaning, and the red triangle reports the ML estimate. The graph is based on 2,500 draws from the joint posterior sample. The posterior distribution of mixture and Markov-switching models: A remark observation is that the MCMC procedures can be used to explore the joint posterior distribution of the model parameters. Specifically, the estimation method is a random-walk Metropolis-Hastings algorithm with a coerced acceptance rate. We observed excellent performance in the context of (identified) mixture models. Using the ML parameter estimates as starting values, we can estimate the model by MCMC.

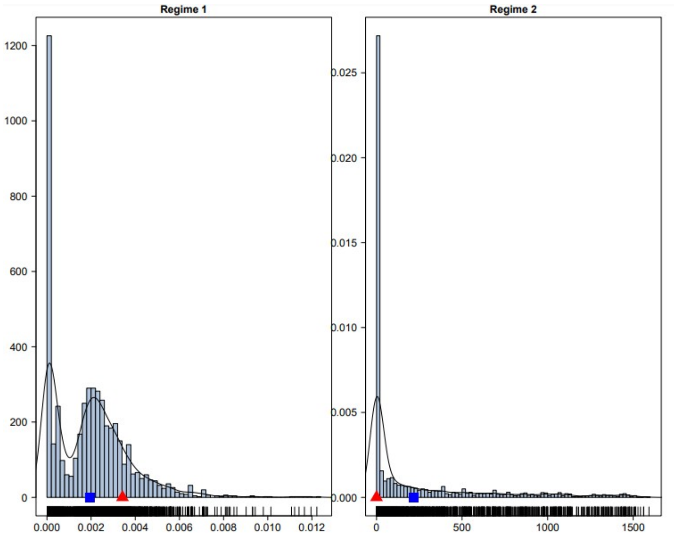

Figure 7: Plot for original y_t to identify any unusual observations

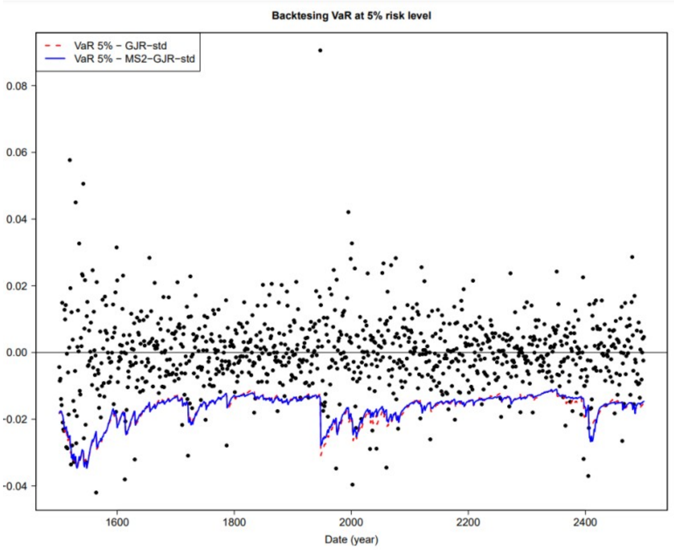

Figure 8. Plot based on the predicted y_t for the next few days

Now, we wish to produce results with 2,500 draws of the posterior sample for the parameters $\alpha_{1,1}$ and $\alpha_{1,2}$. By simulation, we can apply Bayesian estimation to make distributional (probabilistic) statements on any (possibly nonlinear) function of the model parameters. Further, for each drawing in the posterior sample, we can compute the unconditional volatility in each regime, to get its posterior distribution. Figure 7 displays the posterior distributions of the unconditional annualized volatility in each regime. Each blue square reports the posterior meaning while the red triangle reports the ML estimate. Notice that both distributions exhibit positive skewness. We can also extract the posterior draws from an estimated '*MSGARCH − MCMC − FIT'* object and then compute the predictive density for each of them. In addition, we can provide the code for a backtest analysis of the'ms.gjr.s' model against its single-regime counterpart.

```
gjr.s = CreateSpec (variance.spec = list (model =
"gjrGARCH"), distribution.spec = list (distribution = "std"),
switch.spec = list(K = 1))
models = list (gjr.s, ms.gjr.s)
```

Next, we set the backtest settings. We decide to test the performance of the models on 1000 out-of-sample observations and focus on one-step value-at-risk predictions at the 5% risk level. Predictions are based on windows of 1500 observations, and models are recalibrated for 100 observations for frame rate purposes. We initialize a vector for out-of-sample returns and a matrix of VaR forecasts. For each new observation, we use the last 1500 data points to predict the one-step

VaR at the 5% level with a storing. Models are estimated by the ML method every 100 observations in a window out-of-sample. In Figure 8 we display the resulting VaR forecasts of the two models together with the realized returns. The results of this paper are based on simulations of the adaptive MCMC scheme. Therefore the results depend on the value of the *set.seed* and the linear algebra library such the Cholesky decomposition and the eigenvalue calculation used. However, differences may accumulate over MCMC iterations, and so new MCMC results may differ from those reported in this paper.

CONCLUSION

Bayesian modeling is the process of fitting a probability model to a data set and generalizing the result to a (posterior) probability distribution over model parameters and unobserved quantities, such as predictions of new observations. For these purposes, we used (i) the $stan - garch$ function of the Bayesforecast of the R-package in part 1 and (ii) some functions of the MSGARCH package in part 2 and estimated a volatility model of gold prices using daily observations on log-returns. We obtained some useful features of these different models and illustrated their use in analysis to understand how Bayesian estimation differs from traditional approaches. In part 1, we proposed a volatility model for the returns of the gold price series of everyday observations utilizing the stan-garch work of the Bayes forecast package. We utilized the later 2500 observations in phase-1 and 4150 returns in phase-2, to assess the Bayesian GARCH (1, 1) parameters. This study centered on discussions under the presumption of a student's t- distribution and a normal error distribution. Plots for the initial log returns of gold prices and for the predicted series are drawn using an estimated model. In addition, using the posterior distributions of the parameters of the fitted GARCH (1, 1) model, charts for such posteriors are drawn to check the stationery and other related features, In part 2, using the R package MSGARCH, we attempted to estimate, simulate, and perform forecasts with Markov-switching GARCH models. Our focus was to create various single-regime and two-regime-switching specifications with different scedastic functions and conditional distributions. The outcomes of GARCH models assure us of their effectiveness in modelling volatility across various types of financial data and can be expanded to incorporate additional factors that are recognized to affect volatility, to capture the genuine dynamics of financial time series and enhance prediction accuracy.

REFERENCES

Ardia, D. 2008. Financial Risk Management with Bayesian Estimation of GARCH Models. Lecture Notes in Economics and Mathematical Systems 612. *Springer-Verlag, Berlin, Ger- many*. ISBN 978-3-540-78656-6, e-ISBN 978-3-540-78657-3, doi: 10.1007/9783540786573

Ardia, D. 2008. *Lecture Notes in Economics and Mathematical Systems*, Springer-Verlag, volume 612, chapter Financial Risk Management with Bayesian Estimation of GARCH Models: Theory and Applications.

Ardia, D. 2009. Bayesian Estimation of a Markov-Switching Threshold Asymmetric GARCH Model with Student-T Innovations. *Econometrics Journal*, 12(1), pp. 105-126. doi: 10.1111/j.1368-423X.2008. 00253.x

Ardia, D., Bluteau, K., Boudt, K., Catania, L. 2018. Forecasting risk with Markov-switching GARCH models: A large-scale performance study. *Int. J. Forecast*. 34 (4), 733–747. https://doi.org/10.1016/j.ijforecast.2018.05.004

Ardia, D., Hoogerheide, L.F. 2010. Bayesian Estimation of the GARCH (1,1) Model with Student-t Innovations. *R Journal 2*(2), pp.41-47. doi: 10.32614/RJ2010014

Ardia, David and Bluteau, Keven and Boudt, Kris and Catania, Leopoldo and Trottier, Denis- Alexandre 2019. Markov-switching GARCH models in R: The MSGARCH package, *Journal of Statistical Software*, 91,4,

Bollerslev, T. 1986. Generalized autoregressive conditional heteroskedasticity, *Journal of econometrics*, 37, 307–327.

Chen, Hao and Zhang, Jianzhong and Tao, Yubo and Tan, Fenglei 2019. Asymmetric GARCH type models for asymmetric volatility characteristics analysis and wind power forecasting, Protection *and Control of Modern Power Systems*, 4, 4, 1-11

Engle, R. (1982). Autoregressive Conditional Heteroscedasticity with Estimates of the Variance of United Kingdom Inflation. Econometrica, 50(4), 987-1007. url: http://www.jstor.org/stable/1912773.

Engle, R. F. and V. K. Ng 1993. Measuring and testing the impact of news on volatility, *Journal of Finance*, 48, 1749–1778.

Glosten, L.R., Jagannathan, R., Runkle, D.E. 1993. On the relation between the expected value and the volatility of the nominal excess return on stocks. *J. Financ*. 48 (5), 1779–1801. https://doi.org/10.1111/j.1540-6261.1993.tb05128.x

Maciel, Leandro 2021. Cryptocurrencies value-at-risk and expected shortfall: Do regime-switching volatility models improve forecasting? *International Journal of Finance & Economics*, 26, 3, 4840-4855

Makatjane, Katleho and Moroke, Ntebogang 2022. Examining stylized facts and trends of FTSE/JSE TOP40: a parametric and non-parametric approach, *Data Science in Finance and Economics*, 2,3, 294-320

Naik, Nagaraj and Mohan, Biju R. 2021. Stock Price Volatility Estimation Using Regime Switching Technique-Empirical Study on the Indian Stock Market, *Mathematics*,9, 14, 1595-

Oseifuah, Emmanuel K and Korkpoe, Carl H 2019. A Markov regime switching approach to estimating the volatility of Johannesburg stock exchange (JSE) returns, *Investment management and financial innovations*, 16, 1, 215-225

Tan, Chia-Yen and Koh, You-Beng and Ng, Kok-Haur and Ng, Kooi-Huat, 2021. Dynamic volatility modelling of Bitcoin using time-varying transition probability Markov-switching GARCH model,, *The North American Journal of Economics and Finance* 56,101 -377.

Vats, D. and C. Knudson 2020. Revisiting the Gelman-Rubin Diagnostic, *arXiv*, *https://arxiv.org/abs/1812.09384*,

Villa, C. and S. G. Walker 2014. Objective prior for the number of degrees of freedom of a t distribution, *Bayesian Analysis*, 9, 197–220.

Viterbi A 1967. "Error Bounds for Convolutional Codes and an Asymptotically Optimum Decoding Algorithm." *IEEE Transactions on Information Theory,* 13(2), 260–269. doi: 10.1109/tit.1967.1054010

Xiao, Yang 2021. Forecasting extreme risk using regime-switching GARCH models: a case from an energy commodity, *International Journal of Emerging Markets*, 16, 8, 1556-1582

****