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# MIXED CONVECTION BOUNDARY LAYER NANOFLUID FLOW OVER AN INCLINED STRETCHING CYLINDER WITH THERMAL RADIATION

### \*Jarwal, V. K., Choudhary, S. and Sinha, S.

Department of Mathematics, University of Rajasthan, Jaipur-302004, Rajasthan, India

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\*Corresponding author: Jarwal, V. K.,

## ABSTRACT

Lecturers Present article classifies the influence of mixed convection boundary layer *MHD* flow of nanofluid ( $Ag/Cu-H_2O$ ) through porous stretching cylinder in the presence of thermal radiation. The governing non-linear *PDE*'s are reducedto*ODE*'s with boundary conditions by using similarity transformations. The numerical method known as Runge – Kutta fourth order has been taken in to account with shooting technique to solve the obtained *ODE*'s with assisting boundary conditions. The impact of thermal radiation parameter on Nusselt number along with mixed convection parameter and heat generation/absorption parameter are shown by graphical and tabular way. The outcome showed that Nusselt number decreases with an increase in radiation absorption parameter. The comparison of our data with published one has admirable result.

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# **INTRODUCTION**

Nanofluid is mix suspension of nanometer sized solid particles (Cu, Al, Ag, etc.) in base fluid such as water, oil and ethylene glycol, which is first introduced by Choi [4]. Due to the fact that thermal conductivity of the nanoparticles is larger than regular fluids, the term nanofluid has become an interesting topic for the researchers in last few decades. There are several manufacturing and technology applications of nanofluids such as engine cooling, refrigerator, microelectronics, cancer therapy, drug delivery and glues. Heat transfer also uses in various industrial areas such as fuel cells, microelectronics, pasteurization of food and biomedical/ pharmaceutical processes. The steady flow of fluid past over a stretching cylindrical surface was introduced by Wang [27]. Incompressible and laminar viscous fluid flow caused by stretching cylinder was scrutinized by Ishak and Nazar [12]. There are several utilizations of convective flow in a porous medium such as building erection, solar collectors, ventilation procedure and removal of heat from nuclear reactors. Pal *et al.* [22] have conferred the effect of thermal radiation on nanofluid flow with porous medium due to stretching/shrinking sheet. Ishak *et al.* [11] studied the impact of suction/injection on fluid motion and heat transfer towards permeable stretching cylinder. The combined effect of thermal conductivity and dynamic viscosity with the existence of stretching permeable tube in nanofluid was described by Ahmed *et al.* [1].

The concept of magnetohydrodynamics (*MHD*) is very important in devices with high thermal properties like in physiological equipment involving blood pumping machines, *MRI*, hyperthermia and flow in arteries. Ashorynejad *et al.* [2] have studied the impact of *MHD* flow over a stretching cylinder within nanofluid. Majeed *et al.* [14] have scrutinized the mixed influence of partial slip and heat transfer on steady non-NewtonianCassonfluid flow along stretchable cylinder with arranged heat flux. The analytical results of magnetohydrodynamic (*MHD*) third grade fluid flow over a stretching cylinder were computed by Hayat *et al.* [8]. Butt *et al.* [3] investigated the magnetic field effects on entropy generation and viscousflow over a stretching cylinder" was investigated by Hussain *et al.* [10]. Naramgari and Sulochana [21] investigated on "*MHD* flow of dusty nanofluid over a stretching surface with volume fraction of dust particles". Mukhopadhyay [20] proposed *MHD* boundary layer slip flow towards a stretching cylinder. Pandey and Kumar ([23], [25]) have analyzed the boundary layer flow and heat transfer of nanofluid over stretching cylinder with different effects. Malik *et al.* [15] studied the Cattaneo- Christov heat flux model for Sisko fluid flow via nonlinear stretching cylinder. Ohmic-viscous dissipation and heat generation/absorption with magnetic field effect on *Cu*-water nanofluid flow through stretching/shrinking channel was analyzed by Pandey and Kumar [24].

Ganga *et al.* [5] considered "*MHD* radiative boundary layer flow of nanofluid past a vertical plate with internal heat generation/absorption, viscous and ohmic dissipation effects". Sisko nanofluid flow over a stretching surface was analyzed by Khan *et al.* [13] using both analytical and numerical methods in the presence of convective boundary conditions. Second order slip flow of nanofluid over a stretching/shrinking sheet with thermal radiation and magnetic field effect was studied by Hakeem *et al.* [6]. Hayat *et al.* [7] evaluated the role of Joule heating and melting heat transfer in the *MHD* flow of *Cu*-water nanofluid flow. The influence of Joule heating on *MHD* tangent hyperbolic peristaltic nanofluid flow through inclined channel with slip conditions was observed by Hayat *et al.* [9]. Singh *et al.* [26] discussed the non-uniform heat source effects along with melting heat transfer on magnetic *Cu*- water nanofluid flow via porous cylinder. Mishra *et al.* [16]-[19])examined several effects on *MHDAg*-water nanofluid flow utilizing different geometries. In view of above literature survey and development of research in nanofluids, it is revealed that mixed convection *Ag*-water and *Cu* -water nanofluid flow through an inclined porous stretching cylinder with thermal radiation has not been studied yet, therefore the intention of present research is to investigate this aforesaid problem. The governing boundary layer equations are simplified using similarity variables which are then solved numerically using Runge- Kutta fourth order method with shooting technique. The influence of various parameters on heat transfer characteristics and the flow field are explored and depicted through graphs or tables.

#### Nomenclature

$B_0$	Magnetic field strength (m $t^{-2}A^{-1}$ )	$u_w$	Suction velocity (m/s)
$C_p$	Specific heat at constant pressure (J/kg K)	<i>u</i> , <i>w</i>	Velocity component along <i>r</i> - and <i>z</i> - direction (m/s)
Cf	Skin friction coefficient	<i>z</i> , <i>r</i>	Cylindricalcoordinates (m)
D	Local mixed convection parameter		
Ec	Eckert number		Greek symbols
f	Dimensionless stream function	α	Thermal diffusivity (m <sup>2</sup> /s)
g	Acceleration due to gravity $(m/s^{-2})$	β	Thermal expansion coefficient (K <sup>-1</sup> )
$Ha^2$	Magnetic field parameter	γ	Angle of inclination of cylinder
$K_0$	Permeability of the porous medium (m <sup>2</sup> )	К	Thermal conductivity (m <sup>2</sup> /s)
Κ	Permeability parameter	μ	Dynamic viscosity (Kg/ms)
$l_1$	Velocity slip factor (m)	ρ	Density (kg/m <sup>3</sup> )
$l_2$	Thermal slip factor (m)	$\sigma$	Electric conductivity (Kg <sup>-1</sup> m <sup>-3</sup> t <sup>3</sup> A <sup>2</sup> )
Nu	Nusselt number	υ	Kinematic viscosity (m <sup>2</sup> /s)
Pr	Prandtl number	λ	Velocity slip parameter
$Q_0$	Heat generation/absorption coefficient (W/m <sup>2</sup> K)	heta	Dimensionless temperature
Q	Heat generation/absorption parameter	$\delta$	Thermal slip parameter
R	Radiation absorption parameter	$\phi$	volume fraction of nanoparticles
Re	Reynolds number	$\eta$	Similarity variable
S	Suction/injection parameter		Subscript
Т	Temperature of the nanofluid (K)	f	Base fluid
$T_w$	Wall temperature (K)	S	Nanoparticle
$T_{\infty}$	Ambient temperature (K)	nf	Nanofluid

#### **Mathematical Formulation**

We have considered an axisymmetric, incompressible, steady, laminar flow of a nanofluid over a porous inclined stretchingcylinder withdiameter2*a.z.*-axis and *r*-axisare considered in horizontalandvertical direction of the cylinder, respectively as signified in Fig. 1.Towards radial direction we assume that intensity of uniform magnetic field is  $B_0$ . Along z-direction, the cylinder is being stretched with velocity  $W_w = 2cz$ , where *c* is stretching rate. We also assume that  $T_w$  is the surface temperature of the cylinder and ambient fluid temperature is  $T_\infty$ , where  $(T_w > T_\infty)$ . The regular fluid (water) based nanofluid containing Ag (silver) or Cu (copper) as nanoparticles is considered. The physical properties of regular fluid andnano particles are depicted in Table 1. Under the all above postulations, the boundary-layer equations are articulated as follows (Ahmed *et al.* [1], Ashorynejad*et al.* [2] and Ishak *et al.* [11]):



Fig. 1. Physical sketch and the coordinate system

$$\frac{\partial(rw)}{\partial z} + \frac{\partial(ru)}{\partial r} = 0, \tag{1}$$

$$\rho_{nf}\left(w\frac{\partial w}{\partial z}+u\frac{\partial w}{\partial r}\right) = \mu_{nf}\left(\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right) - \sigma_{nf}B_0^2w - \frac{\mu_{nf}w}{K_0} + g\left(\rho\beta\right)_{nf}\left(T-T_{\infty}\right)\cos\gamma,$$
(2)

$$(\rho C_{p})_{nf} \left( w \frac{\partial T}{\partial z} + u \frac{\partial T}{\partial r} \right) = \kappa_{nf} \left( \frac{\partial^{2} T}{\partial r^{2}} + \frac{1}{r} \frac{\partial T}{\partial r} \right) + \sigma_{nf} B_{0}^{2} w^{2} + \mu_{nf} \left( \frac{\partial w}{\partial r} \right)^{2} + Q_{0} \left( T - T_{\infty} \right) - \frac{\partial q_{r}}{\partial r},$$

$$(3)$$

Corresponding boundary conditions for the model are as follows (Mishra and Kumar [16]):

$$u = u_{w}, w = W_{w} + l_{1} \frac{\partial w}{\partial r}, T = T_{w} + l_{2} \frac{\partial T}{\partial r} \quad at \ r = a;$$
  
$$u \to 0, \ T \to T_{\infty} \quad as \quad r \to \infty,$$
  
(4)

where velocity slip factor and thermal slip factor are  $l_1$  and  $l_2$ , respectively.

The effective density  $\rho_{nf}$ , heat capacitance  $(\rho C_p)_{nf}$ , dynamic viscosity  $\mu_{nf}$ , thermal conductivity  $\kappa_{nf}$  and electric conductivity  $\sigma_{nf}$  of nanofluid are described as (Ashorynejad*et al.* [2] and Singh *et al.* [26]):

$$(\rho \beta)_{nf} = (1-\phi)(\rho \beta)_f + \phi(\rho \beta)_s$$

$$\rho_{nf} = (1-\phi)\rho_f + \phi\rho_s, (\rho C_p)_{nf} = (1-\phi)(\rho C_p)_f + \phi(\rho C_p)_s,$$

$$\frac{\mu_{nf}}{\mu_f} = \frac{1}{(1-\phi)^{2.5}}, \frac{\kappa_{nf}}{\kappa_f} = \frac{\kappa_s + 2\kappa_f - 2\phi(\kappa_f - \kappa_s)}{\kappa_s + 2\kappa_f + \phi(\kappa_f - \kappa_s)}, \frac{\sigma_{nf}}{\sigma_f} = (1-\phi) + \phi \frac{\sigma_s}{\sigma_f},$$

$$(5)$$

where volume fraction of nanoparticles is taken as  $\phi$ .

By using Rosseland approximation, the radiative heat flux is

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y},\tag{6}$$

where  $k^*$  is the absorption coefficient,  $\sigma^*$  is the Stefan-Boltzman constant. The temperature difference is assuming such that  $T^4$  may be expanded as a Taylor series about  $T_{\infty}$  and neglecting higher order terms, we get  $T^4 \approx 4T_{\infty}^3 T - 3T_{\infty}^4$  and using it in equation (6), we get

$$q_r = -\frac{16\sigma^* T_{\infty}^3}{3k^*} \frac{\partial T}{\partial y} \qquad \text{and hence } \frac{\partial q_r}{\partial y} = -\frac{16\sigma^* T_{\infty}^3}{3k^*} \frac{\partial^2 T}{\partial y^2}.$$
(7)

To convert the governing equations into the system of nonlinear ODE's, we introduce following similarity variables (Ahmed et. al. [1], Ashorynejadet. al. [2] and Ishak et. al. [11]):

$$u = -\frac{c a}{\sqrt{\eta}} f(\eta), \quad w = 2z c f'(\eta), \quad \eta = \left(\frac{r}{a}\right)^2, \quad \theta(\eta) = \frac{T - T_{\infty}}{T_w - T_{\infty}}, \tag{8}$$

Using Equations (5)-(8) into Equations (2)-(3), we acquire following system of nonlinear ODE's:

$$A_{3}(\eta f''' + f'') + A_{1} \operatorname{Re}(f f'' - f'^{2}) - A_{5} Ha^{2} f' - A_{3} K f' + A_{6} D \cos \gamma \theta = 0,$$
(9)

$$A_{2}\operatorname{Re}\operatorname{Pr} f \theta' + A_{4}(\eta \theta'' + \theta') + A_{3}\operatorname{Pr} Ec \eta f''^{2} + \frac{A_{5}}{2}Ha^{2} Ec \operatorname{Pr} f'^{2} + \frac{4}{3}R\eta \theta'' + Q\theta = 0,$$
(10)

with the association of boundary conditions in terms of f and  $\theta$ :

$$f(1) = S, f'(1) = 1 + \lambda f''(1), \theta(1) = 1 + \delta \theta'(1) \text{ at } \eta = 1;$$

$$f'(\infty) \to 0, \ \theta(\infty) \to 0 \ as \ \eta \to \infty,$$
<sup>(11)</sup>

where prime denotes differentiation with respect to  $\eta$  .

The parameters used in equations (9) to (11) are as follows:

$$\Pr = \frac{\upsilon_{f}}{\alpha_{f}}, \ \operatorname{Re} = \frac{c \, a^{2}}{2\upsilon_{f}}, \ M = \frac{\sigma_{f} B_{0}^{2} \, a^{2}}{2\rho_{f} \upsilon_{f}}, \ K = \frac{a^{2}}{2K_{0}}, \ D = \frac{\beta_{f} g \, a^{2} \left(T_{w} - T_{w}\right)}{2W_{w} \upsilon_{f}}, \ R = \frac{4T_{w}^{3} \, \sigma^{*}}{k^{*} \kappa_{f}}, \\ Q = \frac{Q_{0} \, a^{2}}{4\kappa_{f}}, \ S = -\frac{u_{w}}{c \, a}, \ Ec = \frac{W_{w}^{2} \, \rho_{f}}{\left(\rho C_{p}\right)_{f} \left(T_{w} - T_{w}\right)}, \ \lambda = \frac{2l_{1}}{a}, \ \delta = \frac{2l_{2}}{a}, \ A_{1} = (1 - \phi) + \phi \left(\frac{\rho_{s}}{\rho_{f}}\right), \\ A_{2} = (1 - \phi) + \phi \left(\frac{\left(\rho C_{p}\right)_{s}}{\left(\rho C_{p}\right)_{f}}\right), \ A_{3} = \frac{\mu_{nf}}{\mu_{f}}, \ A_{4} = \frac{\kappa_{nf}}{\kappa_{f}}, \ A_{5} = \frac{\sigma_{nf}}{\sigma_{f}}, \ A_{6} = (1 - \phi) + \phi \left(\frac{\left(\rho\beta\right)_{s}}{\left(\rho\beta\right)_{f}}\right).$$
(12)

Here  $A_1, A_2, A_3, A_4, A_5$  and  $A_6$  are constants.

Important non-dimensional physical quantities, the skin friction coefficient and the local Nusselt number can be defined respectively as follows:

$$C_{f} = \frac{2\mu_{nf}}{W_{w}^{2}\rho_{f}} \left(\frac{\partial w}{\partial r}\right)_{r=a}, \quad Nu = -\frac{a\kappa_{nf}}{\kappa_{f}\left(T_{w} - T_{\infty}\right)} \left(\frac{\partial T}{\partial r}\right)_{r=a},$$
(13)

Now, using Equation (4) and (8) in Equation (13), the reduced skin friction coefficient and the reduced Nusselt number are given as follows:

$$(z \operatorname{Re}/a)C_f = A_3 f''(1), \quad Nu = -2A_4 \theta'(1).$$
 (14)

#### Numerical Method

The dimensionless momentum Equation (9) and energy Equation (10) together with associatedboundary conditions (11) aresolved numerically by applying Runge–Kutta fourth order method with shooting technique. In this problem, we set up the following terms into the obtained boundary layer equations to covertthese equations into first-order *ODE*'s:

$$f = f_{1,} f_{1}' = f_{2,} f_{1}'' = f_{3,} \theta = f_{4}, f_{4}' = f_{5}.$$

Using above mentioned substitutions, the followingsystem of first-order ODE's is obtained:

$$\begin{aligned} f_1' &= f_2, \\ f_2' &= f_3, \\ f_3' &= \left(\frac{1}{\eta A_3}\right) \left[A_1 \operatorname{Re}\left(f_2^2 - f_1 f_3\right) + A_5 H a^2 f_2 + A_3 K f_2 - A_6 D \cos \gamma f_4 - A_3 f_3\right], \\ f_4' &= f_5, \\ f_5' &= -\left\{\frac{1}{\eta \left(A_4 + \frac{4R}{3}\right)}\right\} \left[A_4 f_5 + A_2 \operatorname{Re} \operatorname{Pr} f_1 f_5 + A_3 \eta \operatorname{Pr} \operatorname{Ec} f_3^2 + \frac{A_5}{2} H a^2 \operatorname{Ec} \operatorname{Pr} f_2^2 + Q f_4\right]. \end{aligned}$$

with initial conditions

$$\begin{split} \eta = 1: \ f_1 = S, \ f_2 = 1 + \lambda f_3, \ f_4 = 1 + \delta f_5; \\ \eta \to \infty: \ f_2 \to 0, \ f_4 \to 0. \end{split}$$

Using Runge-kutta fourth order method, the system of first order *ODE*'s along with initial conditions solved and also apply appropriate guessing for missing initial values by the shooting method for several sets of parameters until the conditions  $f'(\eta \rightarrow \infty) = 0$  and  $\theta(\eta \rightarrow \infty) = 0$  hold.

# **RESULTS AND DISCUSSION**

The altered non-dimensional ODE's (9) and (10) with related boundary conditions (11) are solved numerically with the aid of Runge-Kutta fourth order method via shooting technique. We select the range of acting parameters as:  $0.0 \le K \le 4.0$ ,  $1.0 \le Ha^2 \times 10^{-8} \le 4.0$ ,  $-2.0 \le Q \le 2.0$ ,  $0.2 \le \lambda \le 0.4$ and  $0.1 \le \delta \le 0.3$  . Throughout the whole process of numerical analysis, the default values of parameters are considered as: K=1.0,  $Ha^2 \times 10^{-8}=1.0$ , *Ec*=0.3, Pr=6.2, *Q*=0.5, *D*=0.1, *S*=0.1, Re=5.0,  $\delta$ =0.1, R=0.1,  $\phi$ =0.05,  $\gamma = \pi/4$  and  $\lambda$ =0.2 unless stated separately. In Fig. 2 and Fig. 3 consequence of permeability parameter (K) on velocity and temperature profiles are disclosed. It exhibits that velocity profiled iminishes while temperature profile increases with an enhancement in the permeability parameter (K). The variationsamong velocity and temperature profiles for various values of magnetic field parameter ( $Ha^2$ ) are sketched in Fig. 4 and Fig. 5. It is seen from Fig. 4 and Fig. 5 that for increasing value of magnetic field parameter ( $Ha^2$ ), velocity of existing fluid decelerates and temperature accelerate. A force determined as the Lorentz force is always generated for an electrically conducting fluid in the presence of magnetic field. This force reduces the motion of fluid within the boundary layer region. Fig. 6 displays the effect of heat generation/absorption parameter (O) on temperature profile and declares that temperature of nanofluid is an increasing function of Q. The impact of radiation absorption parameter (R)on temperature distribution is portrayed in Fig. 7. From this outline, it is observed that nanofluid temperature increases with the influence of R. Fig. 8 and Fig. 9 are graphed to examine the nature of velocity and temperature of nanofluid corresponding distinct values of Reynolds number (Re). It is cleared from Fig. 8 and Fig. 9 that on accelerating the values of Reynolds number, velocity and temperature of nanofluid continuously decreases. The impact of mixed convection parameter (D) on velocitygraphis shown in Fig. 10.



It is observed from Fig. 10that the velocity boundary thickness of nanofluid rises up with growing values of D. The temperature distribution due to several values of Eckert number (*Ec*) is sketched in Fig. 11. According to this figure, nanofluid temperature accelerates due to an enhancement in Eckert number. Fig. 12 and Fig. 13 depict the velocity and temperature profiles corresponding to the suction/injection parameter (S). These curves identified that velocity and temperature of nanofluid retards as parameter S moves from injection to suction. Fig. 14 shows the influence of

velocity slip parameter ( $\lambda$ ) on velocity profile. This graph declares that as  $\lambda$  increases, the velocity profile decreases. The variation in temperature profile with respect to thermal slip parameter ( $\delta$ ) is depicted in Fig. 15.



Fig. 6. Temperature behavior for various values of Q



Fig. 8. Velocity behavior for various values of Re



Fig. 7. Temperature behavior for various values of R



Fig. 9. Temperature behavior for various values of Re





Fig. 11. Temperature behavior for various values of Ec

From this figure, it is outlined that an enhancement in scauses decrease in temperature profile. Fig. 16 illustrates the effect of inclination angle on velocity of nanofluid. Referring to Fig. 16, we deduce that velocity of nanofluid continuously diminishes when cylinder shifts its position from vertical to horizontal. To examine the authenticity of numerical outcomes, we studied a comparison of present data with published one which is obtained by wang [27], Ahmed et al. [1] and Mishra and Kumar [16]. Admirable results have been detected on viewing Table 2.

0.6

0.5

0.4 ( <sup>1</sup>/<sub>4</sub>), j

0.3

0.2

0.1

0

1



Fig. 12. Velocity behavior for various values of S



Fig. 14. Velocity behavior for various values of  $\lambda$ 



Fig. 16. Velocity behavior for various values of  $\gamma$  when D=5.0



Fig. 13. Temperature behavior for various values of S



Fig. 15. Temperature behavior for various values of  $\delta$ 



Fig. 17. Comparison of Nusselt number for  $Ag-H_2O$  and  $Cu-H_2O$  nanofluid w. r. t. K.

Table 1. Thermophysical properties of water and nanoparticles (Ahmed et. al. [1])

	ho (kg/m <sup>3</sup> )	$C_p$ (J/kg K)	<b>К</b> (W/m K)	$\sigma$ (S/m)	$\beta \times 10^{-5} (\text{K}^{-1})$
Pure water	997.1	4179	0.613	0.05	21
Silver (Ag)	Silver ( <i>Ag</i> ) 10,500 2.		429	$6.3 \times 10^{7}$	1.84
Copper (Cu)	8933	385	401	$5.96 \times 10^{7}$	1.67



Fig. 18. Comparison of Nusselt number for *Ag-H*<sub>2</sub>*O* and *Cu-H*<sub>2</sub>*O* nanofluid w. r. t. *D*.



Fig. 20. Comparison of Nusselt number for *Ag-H*<sub>2</sub>*O* and *Cu-H*<sub>2</sub>*O* nanofluid w. r. t. *Q*.



Fig. 22. Comparison of Nusselt number for  $Ag-H_2O$  and  $Cu-H_2O$  nanofluid w. r. t.  $Ha^2$ 



Fig.19. Comparison of Nusselt number for Ag-H<sub>2</sub>O and Cu-H<sub>2</sub>O nanofluid w. r. t. R.



Fig. 21. Comparison of Nusselt number for *Ag-H*<sub>2</sub>*O* and *Cu-H*<sub>2</sub>*O* nanofluid w. r. t. *Re*.



Fig. 23. Comparison of Nusselt number for  $Ag-H_2O$  and  $Cu-H_2O$  nanofluid w. r. t.  $\delta$ 

Table 2. Comparison of numerous values of  $(-\theta'(1))$  with several values of Prandtl number for  $\phi=0$ , when Re=10,  $\gamma = (\pi/2)$ ,  $\lambda = \delta = Ha^2 = Ec = Q = R = K = D = S = 0$ :

Pr	Wang [30]	Ahmed et al. [1]	Mishra & Kumar [17]	Present study
0.7	1.568	1.58679	1.586790	1.586790
7.0	6.160	6.15776	6.155812	6.155812

Table 3 displays variation in Nusselt number for different numerical values of the parameters used in the current study. By analyzing Table 3, we demonstrate that the Nusselt number for Ag-water and Cu-water nanofluids retards for increasing value of parameters K,  $Ha^2$ , R, Q and  $\delta$ , while opposite trend is found for the parameters D and Re. The above results can also be verified by Figs. 17-23. In addition, we elucidate by referring Figs. 17-23 that heat transfer rate for Cu- $H_2O$  nanofluid is higher than Ag- $H_2O$  nanofluid always.

K	Ha <sup>2</sup> ×10 <sup>-8</sup>	δ	D	R	Re	Q	Nu(Cu)	Nu (Ag)
0.0	1.0	0.1	0.1	0.1	5.0	0.5	5.764110	5.664443
1.0	1.0	0.1	0.1	0.1	5.0	0.5	5.609841	5.515259
4.0							5.251337	5.166946
1.0	2.0	0.1	0.1	0.1	5.0	0.5	5.495331	5.397851
	4.0						5.291876	5.190060
1.0	1.0	0.2	0.1	0.1	5.0	0.5	4.313732	4.251058
		0.3					3.504193	3.458411
1.0	1.0	0.1	2.0	0.1	5.0	0.5	5.655733	5.611874
			5.0				5.826665	5.739297
1.0	1.0	0.1	0.1	0.0	5.0	0.5	6.015532	5.919324
				0.2			5.252656	5.159656
1.0	1.0	0.1	0.1	0.1	6.0	0.5	6.283525	6.186564
					7.0		6.885073	6.786152
1.0	1.0	0.1	0.1	0.1	5.0	-2.0	6.051172	5.969058
						2.0	5.305144	5.200377

Table 3. Numerical values of Nusselt number (Nu), when Ec=0.3, S=0.1,  $\lambda$  =0.2,  $\phi$  = 0.05 and  $\gamma$  =  $\pi/4$ 

# CONCLUSIONS

In this article, we analyze and study the behavior of mixed convection boundary layer *MHD* flow of nanofluid ( $Ag / Cu-H_2O$ ) through porous stretching inclined cylinder in the presence of thermal radiation. Runge-Kutta forth order method with shooting technique in *MATLAB* has been taken in to account to acquire solution of non-linear *ODE*'s with associated boundary conditions. Some major conclusions are summarized as follows:

- > Ag-water and Cu-water nanofluid velocity decreases with enhancement in the permeability parameter and suction/injection parameter.
- > On accelerating the values of Reynolds number and suction/injection parameters, thermal boundary layer of Ag-water and Cu-water nanofluid decelerates.
- > Nusselt number reduces with increasing values of permeability parameter, heat generation/absorption parameter and radiation absorption parameter but it is proportional to mixed convection parameter and Reynolds number for both *Ag*-water and *Cu*-water nanofluids.
- > A larger Nusselt number corresponds to more active convection; here Cu-water nanofluid is more convective than Ag-water nanofluid.

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