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# INFLUENCE OF MOUNTING FITS ON THE EFFECTIVE CLEARANCES AND STRESSES IN BALL BEARINGS

***[1]Badr H. Bedairi, [2]Ahmed M. El-Sherbeeny and [1]Badr S. Azzam**

College of Engineering, Taibah University, Madinah Al Mounawarra (KSA)
College of Engineering, King Saud University, Riyadh (KSA)

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****Corresponding author:*** **Badr H. Bedairi**

**ABSTRACT**

Bearings are considered among the most important elements of power transmission in machines. The main purpose of using bearings is to carry or support rotating shafts in machines to reduce friction and transmit the applied loads from the shafts to the housings. They are classified as either sliding or rolling bearings. Rolling bearings are more efficient, and therefore much more widely used in various engineering applications than other bearing types due to their lower friction, lower maintenance, and more indication signals before failure. There are several parameters that affect the performance of rolling bearings, including type of mounting fits between the bearing and its shaft (and housing), type of lubricating medium, applied loads and types (radial or axial), and the temperature of the environment. The research conducted in this paper has focused on investigating the effect of the mounting fits between the bearing and both the shaft and housing on the bearing performance. The study has examined the effect of different mounting fits on the deformations and stresses induced in the bearing elements and the vibration raised in the bearing during its operation. High stresses and/or high vibration may cause a rapid failure for the bearing, thus, decreasing its working life below the expected or rating life of the bearings.



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## INTRODUCTION

Rolling bearings are considered among the most important elements that are found in almost all rotary machines. They are used for supporting rotating shafts to reduce friction and wear in these rotating parts. These bearings are composed of three main parts, including inner rings, outer rings and rolling elements. The design of these bearings depends on several parameters. Some of these parameters are related to the bearing loads (radial, thrust or combined). While, the others are related to bearing materials and bearing mounting (fits, and assembly and disassembly methods). The mounting fits between the bearing rings and both the shafts and housing are considered to be the main parameter affecting the performance of these bearings. This parameter affects the clearance between the rolling elements and rings, there by affecting the bearing life due to the fiction, wear and vibration which may increase in the bearing due to the wrong fits. The mounting fit may also affect the stresses and load distribution induced in the bearing components.

Hertz [1] was the first researcher who studied the contact stress between bearing rolling elements and rings. He has considered rolling elements as spheres resting on flat surfaces and studied the maximum contact stress and deflection resulting in both the rolling elements and rings. Stribeck [2] presented a mathematical model to study the effect of the maximum normal load characteristics of the bearing on the number of loaded rolling elements in the bearing and resulting contact angle. His mathematical model manipulated the bearing from different relevant aspects, including durability, stiffness, reliability and stability. Tatjana Lazovic et al [3] developed a mathematical model for the load distribution between rolling elements in the ball bearing. The model generates a load distribution function pertaining to all variables with relevant influence on rolling bearing components, including the dimensions of the bearing rings and their raceways, the number of rolling elements, the internal effective clearance, and the external load applied on the bearing. Fred B. et al [4] studied the effect of internal diametral clearance on the fatigue life of both the ball and cylindrical bearings. The load distribution in bearings rolling elements have been investigated for both types of bearings. The load at its heavist value has been described in terms of the Stribeck Number. This is a value that could be used to find the load distribution among rolling elements in a bearing. They developed empirical equations for calculating the maximum loads applied on the rolling-elements and the life factors in both low-speed cylindrical roller bearings and deep-groove ball bearings. S. Li et al [5] submitted a mathematical model for the contact analysis of rolling bearings based on the principles of the mathematical programming method. His model consisted of building a 3D-FEM to calculate the deformation influence coefficients and gaps between assumed pairs of contact points among contact surfaces. They found that for deep groove ball bearings, the calculated contact pressures on ball surface are more reasonable and accurate than the ones obtained by other commercial CAE software. For cylindrical bearings, it was found that edge-loads (non-Hertz contact that cannot be analyzed by Hertz theory) on the two ends of the roller surface were analyzed successfully by the method presented in their paper, provided that the rollers were not crowned longitudinally.

Vidyasagar et al [6] presented a finite element analysis of integral shaft bearings for high load capacities aimed at eliminating the misalignment defects which may cause undesirable distortions in bearings. They presented a numerical analysis for the integral /bearing shafts showing the heat flow, temperature distribution and stresses induced in the bearing system. They concluded that the maximum stresses induced in shafts are within the safe zone of its material strengths. They also concluded that the deformations created in the bearing are within the allowable clearance limits of its rolling elements with rings. Viramgama et al [7] presented an analysis for single row deep groove balls. In their paper, they analyzed the ball bearing using finite element analysis to get the stress level and displacement created in ball bearings. The objective of their work was to find the most influential parameters for radial stiffness of the bearings under an axial load. Throughout their analysis, they conducted a static and dynamic analysis to analyze the life span of the bearing, as well as its rejection rate and productivity.

## METHODOLOGY

**Equivalent load of the rolling bearings:** It is well known that the loads which are carried by the rolling bearings are transmitted from the shaft to the housing through bearing elements; from the shaft to the inner ring, to the rolling elements, to the outer ring, then, finally, to the supporting housing. The equivalent load acting on the bearing can be determined from the following design equation:

$F_e = XVF_r + YF_a$---------------------------------------------------(1)

Where,
X and Y: are the radial and axial factors, respectively (they depend on the bearing static and dynamic capacities)
V : is the rotational factor = 1 for rotating inner rings and = 1.2 for rotating outer rings
$F_r$ and $F_a$ are the radial and axial loads acting on bearing, respectively.

**Number of rolling elements sustaining the applied loads**

The applied load acting on the bearing cannot be simultaneously sustained by the entire number of ball bearings. It is distributed on a certain number of balls in the bearing. This number can be obtained from the following equation, as shown in Figure (1) [3]:

$$Z_s = 2 * \mathrm{INT}\left[\frac{Z+3}{4}\right] - 1 \quad \text{.....................................(2)}$$

$$n = \frac{Z_s - 1}{2} \quad \text{........................................(3)}$$

where;
$Z_s$: number of rolling elements sustaining the applied loads
Z: total number of rolling elements in the bearing
n: number of loaded balls in one side of the bearing from the vertical axis.

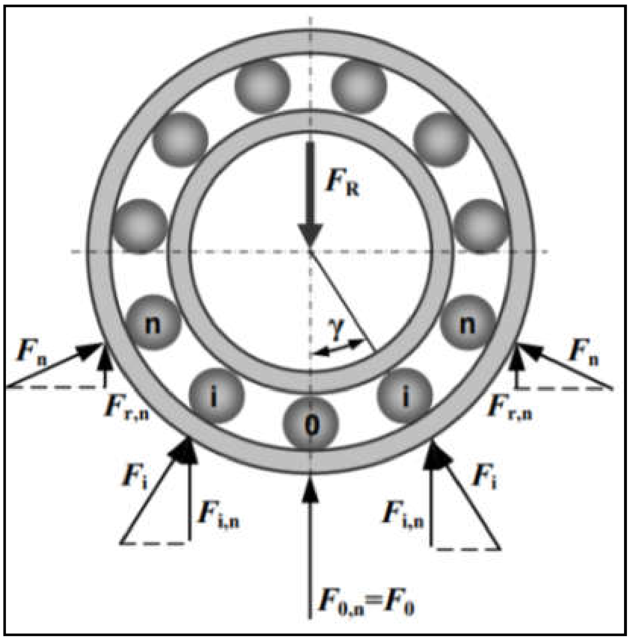


**Figure 1. Load distribution in the ball bearing [3]**

**Load distribution factor in ball bearings**

As previously mentioned, the applied load on the bearing is supported on a certain number of balls that lie on a radial sector space of the bearing. Moreover, this load cannot be carried equally by thoseloaded balls. The degree of inequality of the ball loads depends on a number of factors, which include [3]:

- Applied equivalent load,
- Contact stiffness of the bearing parts (rings and rolling elements)
- Form and accuracy of the bearing parts sizes
- Internal (effective) radial clearance (between the rolling elements and rings).

The sustained load by the loaded balls can be obtained as follows [3]:

$F_i = K_i F_e$ ---------------------------------------------------------(4)

Tatjana et al [13] gave an expression for the radial load distribution factor in an ideal case of load distribution as follows:

$$K_i = \frac{\cos(i\gamma)}{1+2\sum_{j=1}^{n}\cos(j\gamma)}$$

Where;

$F_i$: total (vertical) load sustained by ball "i", as shown in figure (1)

$F_{r,}$ : radial component of the load sustained by ball "i"

$F_e$ : equivalent (total) applied load on bearing

$K_i$ : load distribution factor on ball "i"

$\gamma$ : angle between balls (= 360/Z), Z= total number of balls in bearing

The radial load component applying on a ball at position "i" is:

$F_{r,i}= F_i\cos(i\gamma)$, i= 0,1,….n ……………………………….(6)

**Hertizian stresses in rolling elements and rings**

When two curved bodies are pressed against each other, the contact will be on one point (for balls) or a line (for rollers); which means that the contact area is infinitesimal, and the stresses developed in the two curved bodies will be three-dimensional [8].The most general case of contact stress occurs when each contacting body has a different radius of curvature. Since in ball bearings the contact surface between the balls and their rings is theoretically on points and under radial loads, the induced stress on them will be of Hertizian (contact) type. As a result, the contact stress increases between the ball and rings, and the contact point is reduced into a very small area of radius "a", given by the following equation (7) [8]:

$$a = \sqrt[3]{\frac{3F}{8}\left[\frac{(1-\upsilon_1^2)/E_1 + (1-\upsilon_2^2)/E_2}{1/d_1 + 1/d_2}\right]} \quad \text{………………(7)}$$

Where;

$d_1$ and $d_2$ : diameters of the ball and ring recesses (inner or outer), shown in Figure (2).

a:radius of the contact circular area, shown in figure (2).

$E_1$and $E_2$: elasticity modulus of the ball and ring, respectively.

$\nu_1$and $\nu_2$:Poisson's ratio of the ball and ring, respectively.

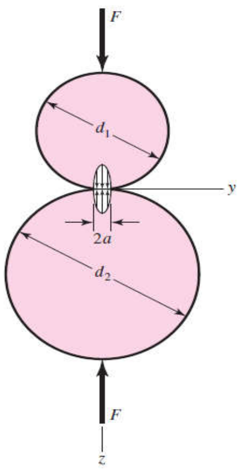


**Figure 2. Contact pressure distribution between two spherical bodies [8]**

The contact pressure distribution within the contact area will be of hemispherical shape with a maximum value $P_{max}$ at the center point of the contact area, as shown in Figure (2).

$$P_{Max.} = \frac{3F}{2\pi a^2} \quad \text{……………………………….(8)}$$

Note that the above equation and Figure 2 show that the magnitude of the maximum stress (pressure) will be below the surface of the ball, which means that the ball failure may start from inside the ball and then propagate until its surface, resulting in wear of pitting type in the bearing. Richard et al. [8] showed that the maximum shear stress is slightly below the surface at a point located at z = 0.48a and is approximated to be of magnitude $0.3P_{max}$. The maximum shear stress is used as a criterion for bearing failure because it is made of carbon steel (a ductile material) and most of steel failures are due to shear stresses.

**Effect of mounting fits on the bearing clearance under loading**

Due to the effect of the mounting fit between the bearing and housing (a transition fit)and between the inner ring and its shaft (a light interference fit), the radial effective clearance between the rolling elements and rings decrease significantly and may be reduced down to zero or even become negative. This internal effective clearance, "δ", shown in figure (3) could be determined using the following equation (9)[9]:

$$\delta = \delta_o - (\delta_{fo} + \delta_{fi} + \delta_t) \text{--------------------------------------(9)}$$

Where,

$\delta_o$: theoretical internal clearance (geometric clearance) between the rings and rolling elements (determined by the manufacturer of the bearing).

$\delta_{fo}$: clearance reduction due to the mounting fit between the outer ring and housing.

$\delta_{fi}$: clearance reduction due to the mounting fit between the inner ring and shaft.

$\delta_t$: clearance reduction due to the temperature difference between the bearing and ambient.

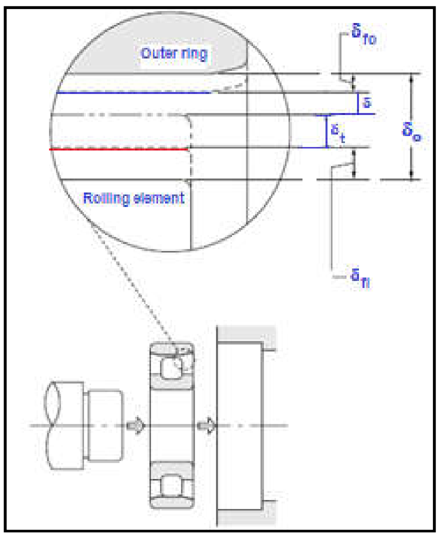


**Figure 3. Radial types of clearances that exist in the bearing [9]**

This zero or negative clearance will be equally distributed among all rolling elements when the bearing is free of loading. However, when the load is applied to the bearing, it causes this zero or negative clearance to increase by an amount equivalent to the deformation raised in the rolling elements due to the loading on one (loaded) side. On the other hand, it will decrease by that same amount on the other (free) side. Therefore, it must be noted that this change in clearance (negative clearance) may increase the contact stresses in rolling elements (for the loaded balls). Meanwhile, on the other side of bearing (free-load side), it may change the effective clearance to excessive positive values, which may cause more vibration for the bearing during operation.

**Effect of applied loading on the effective clearance of the bearing balls**

Due to the applied load, the rolling elements and rings undergo a radial deformation. The relationship between the applied load and resulting deflection is a nonlinear relation, expressed by Eqn. 10 [10]:

$$F = K\,\delta^{m} \quad \text{…………………………………………..(10)}$$

where,
δ: total radial deformation in the bearing due to loading
F: equivalent applied load on bearing
m: constant = 1.5 for ball bearing (= 1.11 for roller bearings)
K: normal stiffness of the rolling bearing material

The normal bearing stiffness depends mainly on the type of rolling elements (balls or rollers), materials of both the rolling elements and rings, the number and size of rolling elements, as well as the contact angle. In Appendix (A), it is explained how to get the value of the normal stiffness of rolling bearings.

**Case Study**

In this case study, a rolling bearing with designation number SKF 6206 single row deep groove ball bearing [7] has been chosen as the study model.

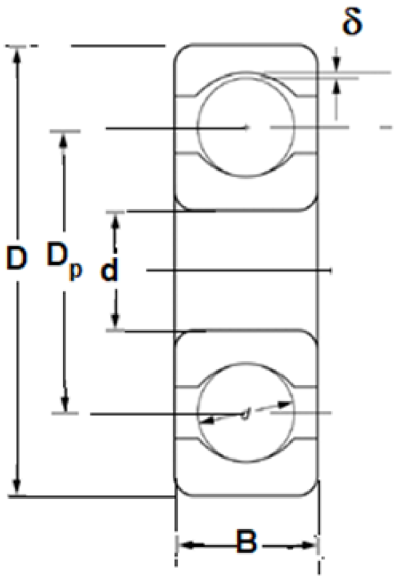


**Figure 4. Deep groove ball bearing dimensions [4]**

**Main dimensions and specifications**

The data of that bearing are shown below, see figure (4) [4]:
Bore (inner diameter) of bearing (d)= 30 mm
Outer diameter of bearings (D)= 62 mm
Pitch diameter ($D_p$)= 46 mm
Ball diameter ($d_b$)= 9.525 mm
Raceway diameter of outer ring= 55.525 mm
Raceway diameter of inner ring= 36.475 mm
Face width (B)= 16 mm
Number of balls= 9 balls
Outer ring and inner ring recess radii ($r_2$)= 5 mm, see figure (5)
Dynamic capacity (C)=20.3 kN
Static capacity ($C_o$)= 11.2 kN

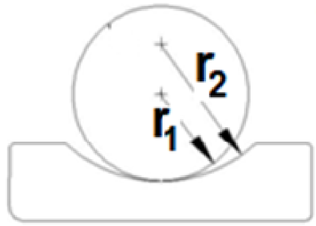


**Figure 5. Cross section of a ball and outer ring recess**

**Bearing materials properties**

The materials of the rings and rolling elements in that bearing is carbon chromium steel containing at least 1% carbon and 1.5% chromium according to ISO 683-17 [SKF]. These elements have undergone a surface induction hardening to produce a high surface hardness in order to sustain the excessive high applied loads with very low wear on their surfaces. The properties of that steel are as follows [6]:
Elasticity modulus= 210 GPa
Ultimate strength=1,570 MPa
Yielding strength=1,370 MPa
Hardness= 62 Rc
Poisson's ratio= 0.3

**Loading data**

The applied radial load is taken as 10 kN without axial load. From equation (1), the equivalent load (Fe) is10 kN (where the load factors are X=1 and Y=0, and the rotating factor isV=1 for the rotating inner race). As mentioned before, this load will not be equally distributed among the loaded balls. he angle spacing the balls is, $\gamma = 360/9 = 40^{o}$. The number of balls that sustained the applied load ($Z_s$)=5 balls, referring to equation (2). The number of loaded balls on one side of the vertical axis (n) = 2 , referring to equation (3). The load distribution factors for$K_0$, $K_1$ and $K_2$are, respectively,0.347, 0.266 and 0.06, referring to equation (5).From equation (4) and Figure (1), one can get the vertical loads sustained by the loaded balls $F_0$,$F_1$ and$F_2$ as3.47 kN, 2.66 kN and 0.6 kN, respectively. Whereas, from equation (6) one can get the radial applied loads on loaded balls $F_{r,0}$ , $F_{r,1}$ and$F_{r,2}$ as3.470kN, 2.038kN and 0.104kN, respectively.

**Stress and deformation results due to the applied loading**

From the above results,it can be shown that (as expected), the rolling element will be subjected to its maximum load when it is in the vertical position (the direction of the applied load). Hence, this rolling element (ball) will be considered as the most critical rolling element as it undergoes the highest contact stress. From equation (7), the radius of the contact area "a" is determined as 1.3125mm. Consequently, the Hertizian (contact) stress is determined from equation (8) to be about962 MPa. This maximum stress will be at the mid-point of the contact zone and it will be at a point radially inside the ball by a distance of 0.63 mm from its surface (about 0.48a). The normal stiffness of that bearing has been also calculated as explained in Appendix A and found to be equals 292,296N/mm$^{1.5}$. By substitution in equation (10), the bearing deformation due to the applied load is found to be as: 0.105mm. This deformation will affect the radial clearance between the bearing elements and rings, where it may increase the negative effective clearance and, therefore, increasing the contact pressure between the rolling elements (in the loaded side). On the other hand, it increases the radial clearance in the unloaded side of bearing, which may in turn increase the vibration amplitude in the bearing during operation. The following subsection will show the effect of different mounting fits on the effective clearance after the bearing has been loaded.

## RESULTS AND DISCUSSION

**Effect of mounting fits on the bearing effective clearance:** As previously explained, due to the mounting fits, the bearing effective clearance will decrease to zero or sometimes to negative values. When the load is applied to the bearing, this effective clearance will change in different manners on the loaded and free sides of the bearing. Table (1) shows the values of that clearance before and after the loading.

**Table 1. The effective clearance for different bearing/shaft fits (before and after loading)**

| Fit designation for inner ring[1] | Effective Clearance due to mounting fits (without loading) | Effective Clearance after loading in the loaded side | Effective Clearance after loading in the free-loaded side |
|---|---|---|---|
| Φ30H7/k5 | -0.008 mm | -0.109 mm | +0.101 mm |
| Φ30H7/m5 | -0.014 mm | -0.112 mm | +0.098 mm |
| Φ30H7/n5 | -0.021 mm | -0.115 mm | +0.095 mm |
| Φ30H7/p5 | -0.030 mm | -0.120 mm | +0.090 mm |

[1] the outer ring/housing fit is Φ62 N8/h7

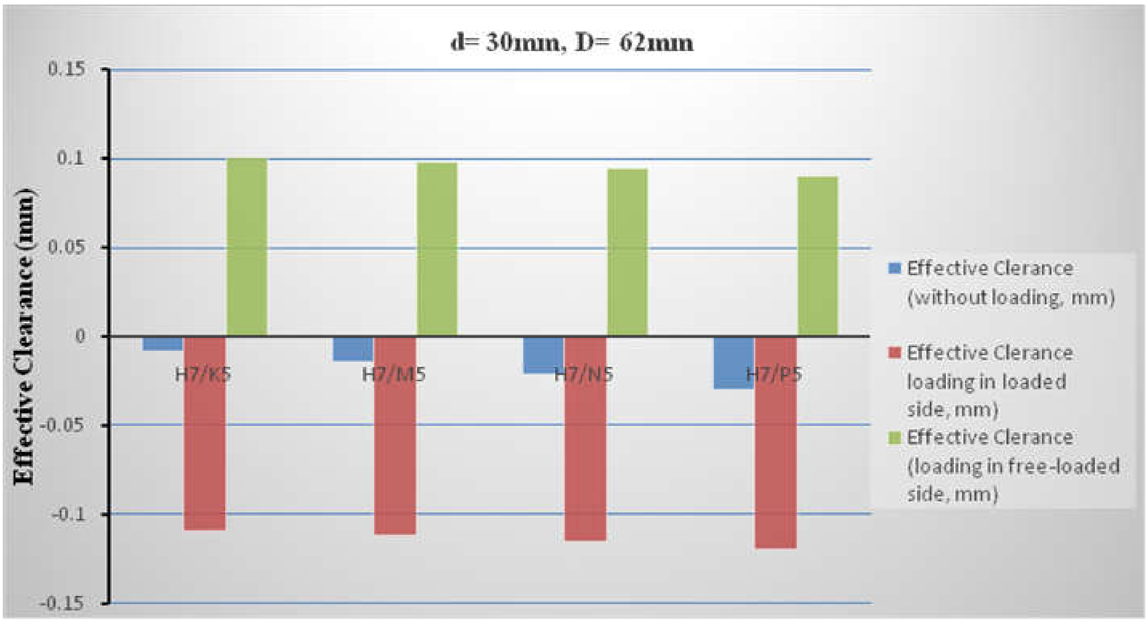


**Figure 6. The effective clearance for different bearing/shaft fits (before and after loading)**

**Table 2. The effective clearance for different bearing/housing fit (before and afterloading)**

| Fit designation for inner ring[2] | Effective Clearance due to mounting fits (without loading) | Effective Clearance after loading in the loaded side | Effective Clearance after loading in the free-loaded side |
|---|---|---|---|
| Φ62 J8/h7 | +0.018 mm | -0.096 mm | +0.114 mm |
| Φ62 K8/h7 | +0.004 mm | -0.103 mm | +0.107 mm |
| Φ62M8/h7 | -0.005 mm | -0.108 mm | +0.103 mm |
| Φ62N8/h7 | -0.014 mm | -0.112 mm | +0.098 mm |

[2] the inner ring/shaft fit is Φ30 H7/m5

**Table 3. The effective clearance for different bearing/shafts fits (before and after loading) with the effect of temperature differences between the bearing and the environment**

| Fit designation for inner ring[3] | Effective Clearance due to mounting fits (without loading) | Effective Clearance after loading in the loaded side | Effective Clearance after loading in the free-loaded side |
|---|---|---|---|
| Φ30H7/ k5 | -0.015mm | -0.112 mm | +0.105 mm |
| Φ30H7/m5 | -0.021 mm | -0.115 mm | +0.095 mm |
| Φ30H7/n5 | -0.028 mm | -0.119 mm | +0.091 mm |
| Φ30H7/p5 | -0.037 mm | -0.124 mm | +0.087 mm |

[3] the outer ring/housing fit is Φ62 N8/h7

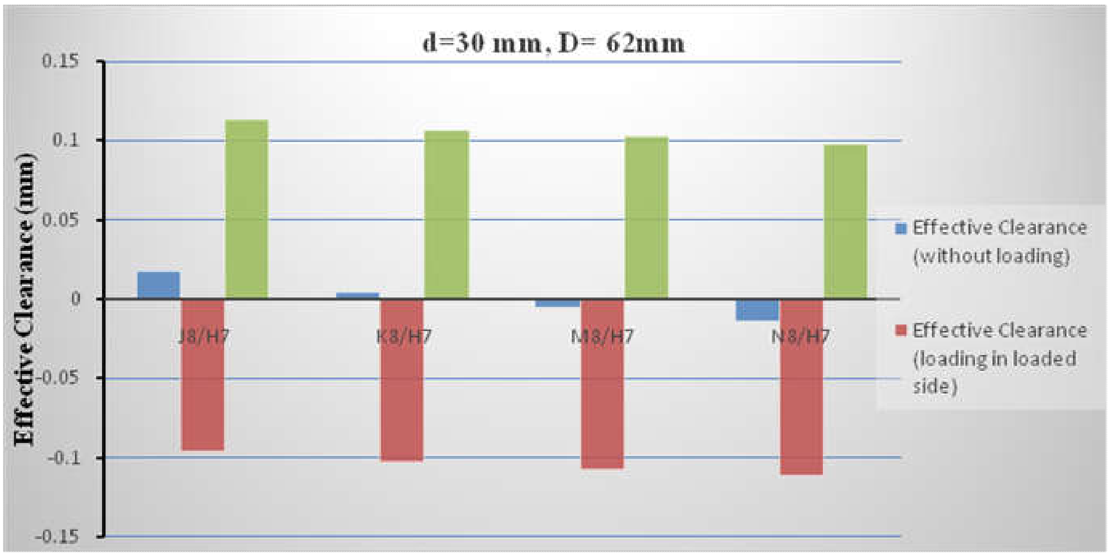


**Figure 7. The effective clearance for different bearing/housing fits (before and after loading)**

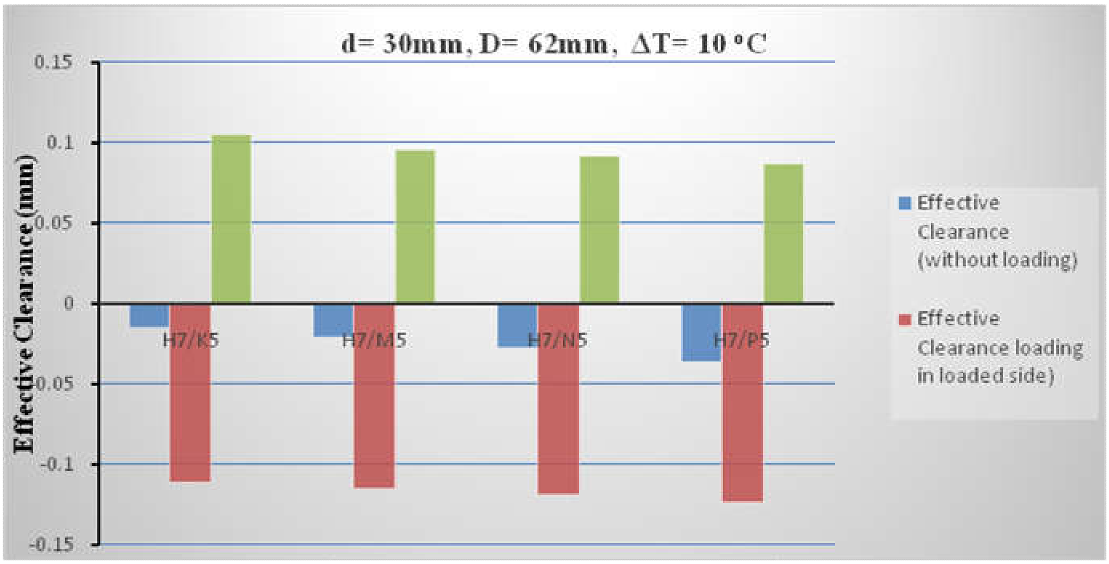


**Figure 8. The effective clearance for different bearing/shaft fits with the effect of temperature differences between bearing and environment**

**Table 4. The effective clearance for different bearing/housing fits (before and after loading) with the effect of temperature differences between the bearing and the environment**

| Fit designation for outer ring[4] | Effective Clearance due to mounting fits (without loading) | Effective Clearance after loading in the loaded side | Effective Clearance after loading in the free-loaded side |
|---|---|---|---|
| Φ62 J8/h7 | +0.003 mm | -0.103 mm | +0.107 mm |
| Φ62 K8/h7 | -0.022 mm | -0.116 mm | +0.096 mm |
| Φ62M8/h7 | -0.020 mm | -0.115 mm | +0.095 mm |
| Φ62N8/h7 | -0.029 mm | -0.120 mm | +0.090 mm |

[4] the inner ring/shaft fit is Φ30 H7/m5

**Table 5. The effective clearance for different bearing/shafts fits with the effect of temperature differences between bearing and environment**

| Temperature difference, $\Delta_t$ | Effective Clearance due to mounting fits (without loading) | Effective Clearance after loading in the loaded side | Effective Clearance after loading in the free-loaded side |
|---|---|---|---|
| 10 ℃ | -0.021 mm | -0.115 mm | +0.095 mm |
| 20 ℃ | -0.028 mm | -0.119 mm | +0.091 mm |
| 30 ℃ | -0.035 mm | -0.122 mm | +0.088 mm |
| 40 ℃ | -0.042 mm | -0.126 mm | +0.084 mm |

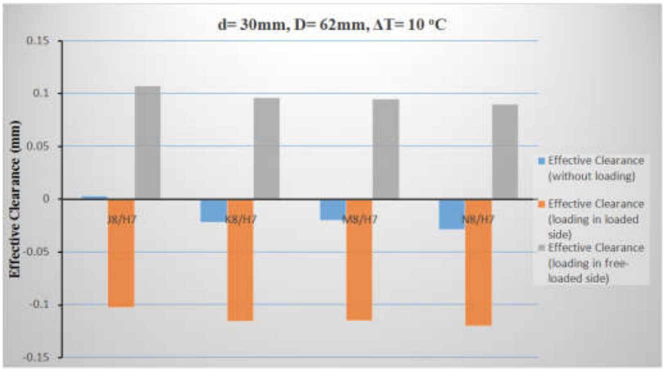


**Figure 9. The effective clearance for different bearing/housing fits (before and after loading) with the effect of temperature differences between bearing and environment**

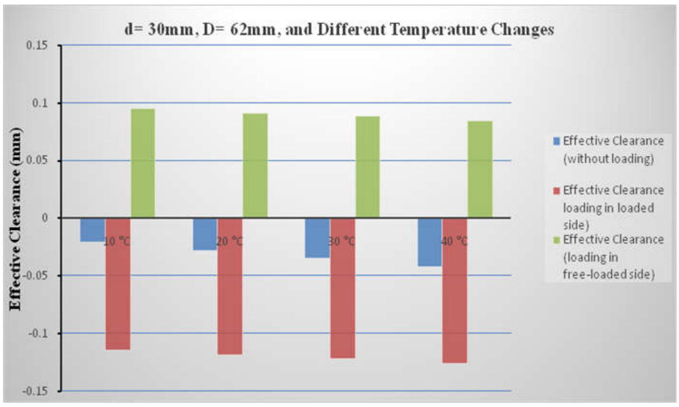


**Figure 10. The effective clearance for different bearing/shafts fits with the effect of temperature differences between bearing and environment**

## DISCUSSION OF RESULTS

The results shown in figure (6) illustrate that the effective clearance between the rolling elements and rings of the bearing in the loaded side will increase negatively with the increase of the degree of the interference fit between the shaft and bearing. This means that the induced stresses in the bearing elements due to the applied loading will increase leading to a decrease in the bearing working life. Conversely, the effective clearance in the free-loaded side will decrease in its positive values, which means that the vibration effect on the bearing during operation will decrease and so the bearing failure due to vibration may be decrease. On the other hand, the type of bearing/shaft fit has minimal effect on the effective clearance of no-load bearings, which cannot affect the bearing assembly and disassembly processes. Similarly, as stated earlier before, the results provided in Figure (7) show that the effective clearance between the rolling elements and rings in the loaded side will increase negatively with the increase in the degree of the transition fit between the bearing and housing. This means that the induced stresses in the bearing elements due to the applied loading will increase leading to a decrease in the bearing working life. On the other hand, the effective clearance in the free-loaded side will decrease in its positive values, which means that the vibration effect on the bearing during operation will decrease, and, thus, the bearing failure due to vibration may be decrease. Contrary to the bearing/shaft fit, however, the type of bearing/housing fit may affect the effective clearance where it changes from positive to negative values, which can affect the bearing assembly and disassembly processes. The results provided in Figures (8 and 9) show that the temperature difference between the bearing and environment has very little effect on the performance of the bearing operation for temperature differences lower than 10 $^{o}$C. This means that at low temperature changes between the bearing and environment, the effect of temperature difference on the bearing performance can be ignored. The results provided in figure 10 show that the high temperature differences between the bearing and the environment can increase the negative effective clearance in the loaded side of the bearing, thereby increasing the induced stresses in the bearing. This may cause a rapid failure for the bearings. On the other hand, the high temperature differences between the bearing and environment can reduce the positive effective clearance in the free-loaded side of bearing, which can reduce the vibration between the bearing elements, there by increasing its working life.

### Conclusions

Throughout the analysis performed in this research paper, the following conclusions could be withdrawn,

- The high degree of interference fit between the bearing and shaft can increase the induced stresses in the bearing, which lead to a decrease in its working life. On the other hand, this high degree of fits can reduce the effective clearance in the bearing, which decreases the vibration raised in the bearing during operation.
- The high degree of transition fit between the bearing and housing can also increase the stresses induced in the bearing and also decrease the vibration raised in the bearing.
- the moderate fit between the bearing and mating parts (shafts and housings) is, therefore, a preferred choice for reducing the stresses and vibration in bearings.
- The high temperature differences between the bearing and environment can affect the bearing performance and also affect the bearing assembly and disassembly processes.

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**Appendix A: Normal Stiffness of Ball Bearing Mounting Fits[4]**

$$K = \left[\frac{1}{k_i^{2/3}} + \frac{1}{k_o^{2/3}}\right]^{\frac{-3}{2}}$$

$$K_o = \pi e_o E_o \sqrt{\frac{2\zeta_o R_o}{9\tau_o^3}}$$

$$K_i = \pi e_i E_i \sqrt{\frac{2\zeta_i R_i}{9\tau_i^3}}$$

where,

K : normal stiffness of the ball bearing
$K_i$ : stiffness of the inner ring
$K_o$ : stiffness of the outer ring
$E_i$ and $E_o$: effective modulus of elasticity for the inner and outer rings materials, respectively.
$e_i$ and $e_o$ : ellipticity parameters of the inner and outer rings, respectively.
$\zeta_\iota$ and $\zeta_o$ : elliptic integral of the second order for the inner and outerrings, respectively.
$\tau_\iota$ and $\tau_o$ : elliptic integral of the first order for the inner and outer rings, respectively.
$R_i$ and $R_o$: effective radius of curvature of the inner and outer rings, respectively.

$$R_o = \left[\frac{2D_p}{d_b(D_p + d_b)} + \frac{2f_o - 1}{f_o d_b}\right]$$

$$R_i = \left[\frac{2D_p}{d_b(D_p - d_b)} + \frac{2f_i - 1}{f_i d_b}\right]$$

Where; $f_i$ and $f_o$ are the race conformity of rings (= $r/d_b$), and r = radius of recess in rings, see figure (5).

an $e_i = (\alpha_i)^{\frac{2}{\pi}}$ $\quad e_o = (\alpha_o)^{\frac{2}{\pi}}$

Where; $\alpha_i$ and $\alpha_o$ are the radius ratios for rings

$$\alpha_i = \frac{2 f_i D_p}{(D_p - d_b)(2f_o - 1)}$$

$$\zeta_i = \frac{\pi}{2} + \left(\frac{\pi}{2} - 1\right)/\ln \alpha_i$$

$$\zeta_o = \frac{\pi}{2} + \left(\frac{\pi}{2} - 1\right)/\ln \alpha_o$$

$$\alpha_o = \frac{2 f_o D_p}{(D_p + d_b)(2f_o - 1)}$$

$$\tau_o = 1 + \frac{(\pi/2) - 1}{\alpha_o}$$

$$\tau_i = 1 + \frac{(\pi/2) - 1}{\alpha_i}$$

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