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IN GENERALIZED RIEMANNIAN GEOMETRY, ROLE OF NON-SYMMETRIC TENSOR $\underset{\vee}{g}_{\mu\nu}$ AS ELECTROMAGNETIC FORCE IN THE UNIFIED FIELD THEORY

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ABSTRACT

Our aim is to get electromagnetic force in generalized Riemannian geometry similar to Maxwell electromagnetic force F_{ij}. For this non-symmetric metric $\underset{\nu}{g}_{ij}$ is transformed by Levi Civita transformation. Transformed $\underset{\vee}{g}_{ij}$ is a pseudo tenser of weight one, but the force obtained is different from Maxwell tensor force F_{ij}. Second transformation was done from $\underset{\vee}{g}_{ij}$ to $\overset{*ij}{\underset{\vee}{g}}$ by taking dual of $\underset{\vee}{g}_{ij}$. We observe that now the force is similar to that of Maxwell force with a difference that of values. Thus a new Maxwell equation was obtained in generalized Riemannian geometry. Using Lorentz force we get the equation of motion of a charge particle in generalized Riemannian geometry which contains the term $\overset{*ij}{\underset{\vee}{g}}$ representing the force.

INTRODUCTION

To unify gravitational and electromagnetic forces Einstein constructed the following principles [10].

- The unify field of gravitation and electromagnetism are determined by sixteen potentials.
- In four dimensional space time, curvature and torsion are determined by these sixteen potentials.
- Imposing certain conditions on curvature and torsion of space time, the set of sixteen potentials is a solution of a system of differential field equation. The identification of gravitational and electromagnetic fields with the potential has to derive from the field equations.

In our paper we agree with these principles. Throughout the paper c=1 is taken where c is the velocity of light. The phenomenon of gravity has described by the Einstein as a geometrical property of space time and it manifests itself through the metric tensor $g_{\mu\nu}$ which is symmetric. The Maxwell's electromagnetic theory which deals with the forces acting on the electrically charged particles cannot be described through the symmetric part of the metric tensor, in general relativity. In Riemannian geometry the non symmetric $g_{\mu\nu}$ vanishes and torsion tensor does not exist. So it was required to generalize Riemannian geometry by introducing non symmetric $\underset{\vee}{g}_{\mu\nu}$ and torsion $\underset{\vee}{\Gamma}^{\alpha}_{\beta\gamma}$. The asymmetric tensor $g_{\mu\nu}$ can be written as the as the sum of symmetric and non-symmetric parts

$$g_{\mu\nu} = \frac{1}{2}\left(g_{\mu\nu} + g_{\nu\mu}\right) + \frac{1}{2}\left(g_{\mu\nu} - g_{\nu\mu}\right)$$
$$= \underset{-}{g}_{\mu\nu} + \underset{\vee}{g}_{\mu\nu}$$

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Where $g_{\underline{\mu\nu}}$ is symmetric part and $g_{\underset{\vee}{\mu\nu}}$ is non-symmetric part. A gravitational field is described by ten components of a symmetric quadratic tensor $g_{\underline{\mu\nu}}$ and the electromagnetic field is described by six components of non-symmetric tensor $g_{\underset{\vee}{\mu\nu}}$. Similarly the displacement tensor $\Gamma^{\alpha}_{\beta\gamma}$ can be split into symmetric and non-symmetric parts.

$$\Gamma^{\alpha}_{\beta\gamma} = \Gamma^{\alpha}_{\underline{\beta\gamma}} + \Gamma^{\alpha}_{\underset{\vee}{\beta\gamma}}$$

Where

$$\Gamma^{\alpha}_{\underset{\vee}{\beta\gamma}} = \frac{1}{2}\left(\Gamma^{\alpha}_{\beta\gamma} - \Gamma^{\alpha}_{\gamma\beta}\right) = S^{\alpha}_{\beta\gamma}$$

$S^{\alpha}_{\beta\gamma}$ is the torsion tensor.

In 1979 Moffat made the observation [5] that the non symmetric part of the generalized metric tensor need not necessary represents electromagnetism, it may represent a new hypothetical force. Later in 1995 he noted that [6] the field corresponding with the non symmetric part need not be mass less like the electromagnetic (or gravitation field). M. W. Evan [7], [8] present the non symmetric metric tensor by the wedge product

$$g^{\mu\nu(A)} = g^{\mu} \wedge g^{\nu}$$

He inferred that if gravitation is identified through $g^{\mu\nu(s)}$ through well known equation

$$R_{\lambda\mu} - \frac{1}{2} g_{\lambda\mu} R = -k(\Gamma_{\lambda\mu})$$

The electromagnetism is defined through $g^{\mu\nu(A)}$.This inference is developed into a general covariant field equation of gravitation and electromagnetism, an equation written in terms of metric four vector q^{μ} which is root of both gravitation and electromagnetism. Papapetrov [3] showed that the gravitational and electromagnetic fields influence each other. Our aim is to get a transformation of nonsymmetric metric tensor which is similar to Maxwell electromagnetic force.

Electromagnetic Potential

According to Einstein [1] from the covariant vector A_i one can form through multiplication, the particular symmetric covariant tensor $A_i A_k$ from such tensor every symmetric tensor of rank two can be obtain through summation with real coefficient

$$g_{ik} = cA_i A_k$$

Since in general theory of relativity g_{ik} represent a gravitational field, A_i can be recognized as the gravitation potential. Every non-symmetric tensor of rank two can be obtained through summation

$$g_{\underset{v}{ij}} = \varepsilon_{ijkl} A_k A_l$$

Where

$$\varepsilon_{ijkl} = \begin{cases} +1 & \text{If(i, j, k, l) is even permutation of } (0,1,2,3) \\ -1 & \text{If(i, j, k, l) is odd permutation of } (0,1,2,3) \\ 0 & \text{If any two suffixes are the same} \end{cases}$$

is Levi Civita symbol

$g_{\underset{\vee}{ij}}$ can be written in the form of 4×4 matrix as follows

$$g_{\underset{\vee}{ij}} = \begin{bmatrix} 0 & A_2A_3 - A_3A_2 & A_3A_1 - A_1A_3 & A_1A_2 - A_2A_1 \\ A_3A_2 - A_2A_3 & 0 & A_0A_3 - A_3A_0 & A_2A_0 - A_0A_2 \\ A_1A_3 - A_3A_1 & A_3A_0 - A_0A_3 & 0 & A_0A_1 - A_1A_0 \\ A_2A_1 - A_1A_2 & A_0A_2 - A_2A_0 & A_1A_0 - A_0A_1 & 0 \end{bmatrix}$$

Due to multiplication by ε_{ijkl}, $g_{\underset{\vee}{ij}}$ will be a weighted tensor of weight one.

$$g_{\underset{\vee}{ij}} \overset{def}{=} A_i A_j - A_j A_i$$

Hence

$$g_{\underset{\vee}{ij}} = \begin{bmatrix} 0 & g_{\underset{\vee}{23}} & g_{\underset{\vee}{31}} & g_{\underset{\vee}{12}} \\ g_{\underset{\vee}{32}} & 0 & g_{\underset{\vee}{03}} & g_{\underset{\vee}{20}} \\ g_{\underset{\vee}{13}} & g_{\underset{\vee}{30}} & 0 & g_{\underset{\vee}{01}} \\ g_{\underset{\vee}{21}} & g_{\underset{\vee}{02}} & g_{\underset{\vee}{10}} & 0 \end{bmatrix}$$

$$g_{\underset{\vee}{ij}} \overset{def}{=} \frac{\partial A_i}{\partial x^j} - \frac{\partial A_j}{\partial x^i}$$

Hence

$$g_{\underset{v}{ij}} = \begin{bmatrix} 0 & \frac{\partial A_2}{\partial x^3} - \frac{\partial A_3}{\partial x^2} & \frac{\partial A_3}{\partial x^1} - \frac{\partial A_1}{\partial x^3} & \frac{\partial A_1}{\partial x^2} - \frac{\partial A_2}{\partial x^1} \\ \frac{\partial A_3}{\partial x^2} - \frac{\partial A_2}{\partial x^3} & 0 & \frac{\partial A_0}{\partial x^3} - \frac{\partial A_3}{\partial x^0} & \frac{\partial A_2}{\partial x^0} - \frac{\partial A_0}{\partial x^2} \\ \frac{\partial A_1}{\partial x^3} - \frac{\partial A_3}{\partial x^1} & \frac{\partial A_3}{\partial x^0} - \frac{\partial A_0}{\partial x^3} & 0 & \frac{\partial A_0}{\partial x^1} - \frac{\partial A_1}{\partial x^0} \\ \frac{\partial A_2}{\partial x^1} - \frac{\partial A_1}{\partial x^2} & \frac{\partial A_0}{\partial x^2} - \frac{\partial A_2}{\partial x^0} & \frac{\partial A_1}{\partial x^0} - \frac{\partial A_0}{\partial x^1} & 0 \end{bmatrix}$$

Using the definition

$$\mathrm{E}x^1 = \frac{\partial \mathrm{A}_1}{\partial x^0} - \frac{\partial \mathrm{A}_0}{\partial x^1} \quad \text{and} \quad \mathrm{B}x^1 = \frac{\partial \mathrm{A}_3}{\partial x^2} - \frac{\partial \mathrm{A}_2}{\partial x^3} \quad \text{etc}$$

We get

$$\mathrm{g}_{\underset{\vee}{ij}} = \begin{bmatrix} 0 & -\mathrm{B}x^1 & -\mathrm{B}x^2 & -\mathrm{B}x^3 \\ \mathrm{B}x^1 & 0 & -\mathrm{E}x^3 & \mathrm{E}x^2 \\ \mathrm{B}x^2 & \mathrm{E}x^3 & 0 & -\mathrm{E}x^1 \\ \mathrm{B}x^3 & -\mathrm{E}x^2 & \mathrm{E}x^1 & 0 \end{bmatrix}$$

We observe that this transformation is different from Maxwell electromagnet force. Taking the dual tensor [11] of the above matrix, we get

$$\mathrm{g}^{\underset{\vee}{*ij}} = \sqrt{\mathrm{g}} \begin{bmatrix} 0 & \mathrm{g}^{\overset{10}{\vee}} & \mathrm{g}^{\overset{20}{\vee}} & \mathrm{g}^{\overset{30}{\vee}} \\ -\mathrm{g}^{\overset{10}{\vee}} & 0 & -\mathrm{g}^{12} & \mathrm{g}^{13} \\ -\mathrm{g}^{\overset{20}{\vee}} & \mathrm{g}^{\overset{12}{\vee}} & 0 & -\mathrm{g}^{23} \\ -\mathrm{g}^{\overset{30}{\vee}} & -\mathrm{g}^{\overset{31}{\vee}} & \mathrm{g}^{03} & 0 \end{bmatrix}$$

Or

$$\mathrm{g}^{\underset{\vee}{*ij}} = \sqrt{\mathrm{g}} \begin{bmatrix} 0 & \mathrm{E}x^1 & \mathrm{E}x^2 & \mathrm{E}x^3 \\ -\mathrm{E}x^1 & 0 & -\mathrm{B}x^3 & \mathrm{B}x^2 \\ -\mathrm{E}x^2 & \mathrm{B}x^3 & 0 & -\mathrm{B}x^1 \\ -\mathrm{E}x^3 & -\mathrm{B}x^2 & \mathrm{B}x^1 & 0 \end{bmatrix}$$

We observe that $\mathrm{g}^{\underset{\vee}{*ij}}$ is a weighted tensor of weight two.

We also observe that the transformation is similar to Maxwell electromagnetic force. Hence in generalized Riemannian geometry the equation analogous to Maxwell equation will be

$$\frac{\partial \mathrm{g}^{\underset{\vee}{*ij}}}{\partial x^j} = \mathrm{J}^i$$

And

$$\frac{\partial \mathrm{g}^{\underset{\vee}{*ij}}}{\partial x^k} + \frac{\partial \mathrm{g}^{\underset{\vee}{*jk}}}{\partial x^i} + \frac{\partial \mathrm{g}^{\underset{\vee}{*ki}}}{\partial x^j} = 0$$

In the Generalized Riemannian Geometry Equation of Electromagnetic Field

The equation of the gravitational field is

$$\mathrm{R}_{\lambda\mu} - \frac{1}{2} g_{\lambda\mu}\mathrm{R} = -k(\Gamma_{\lambda\mu})$$

Let $\Gamma_{\lambda\mu}$ be asymmetric tensor, hence it can be written as the sum of symmetric and non symmetric parts

$$\mathrm{R}_{\lambda\mu} - \frac{1}{2} g_{\lambda\mu}\mathrm{R} = -k(\Gamma_{\underset{-}{\lambda\mu}} + \Gamma_{\underset{\vee}{\lambda\mu}})$$

In generalized Riemannian geometry, the torsion tensor exist, hence the non symmetric part of $R_{\lambda\mu}$ will also exist

$$\left(\mathrm{R}_{\underset{-}{\lambda\mu}} + \mathrm{R}_{\underset{\vee}{\lambda\mu}}\right) - \frac{1}{2}\left(g_{\underset{-}{\lambda\mu}} + g_{\underset{\vee}{\lambda\mu}}\right)\mathrm{R} + k\left(\Gamma_{\underset{-}{\lambda\mu}} + \Gamma_{\underset{\vee}{\lambda\mu}}\right) = 0$$

Separating symmetric and antisymmetric part we get

$$\mathrm{R}_{\underset{-}{\lambda\mu}} - \frac{1}{2} g_{\underset{-}{\lambda\mu}}\mathrm{R} + k\Gamma_{\underset{-}{\lambda\mu}} = 0$$

And

$$\mathrm{R}_{\underset{\vee}{\lambda\mu}} - \frac{1}{2} g_{\underset{\vee}{\lambda\mu}}\mathrm{R} - 2\alpha\Gamma_{\underset{\vee}{\lambda\mu}} = 0$$

Let $\alpha = -\frac{k}{2}$ [5], where α is the electro-magnetic fine structure constant.

The above two equations are equations of gravitational and electromagnetic fields respectively

The component Γ_{00} of stress energy tensor, $\Gamma_{\underset{-}{\lambda\mu}}$ represent energy density.

The components $\Gamma_{\lambda 0}$ for $\lambda = 1,2,3$ represent momentum density

$\Gamma_{\lambda\lambda}$ for $\lambda = 1,2,3$ represent pressure.

$\Gamma_{0\lambda}$ for $\lambda = 1,2,3$ represent energy flux.

$\Gamma_{\underset{-}{12}}, \Gamma_{\underset{-}{13}}, \Gamma_{\underset{-}{23}}$ Represent viscosity.

$\Gamma_{\underset{-}{21}}, \Gamma_{\underset{-}{31}}, \Gamma_{\underset{-}{32}}$ Represent momentum flux.

For electromagnetic stress energy tensor $\Gamma_{\underset{\vee}{\lambda\mu}}$ .The components $\Gamma_{\underset{\vee}{0\mu}}$ represent the flow of electromagnetic energy where $\mu = 1,2,3$ .The components $\Gamma_{\underset{\vee}{\lambda 0}}$ represent the momentum density of electromagnetic field for $\lambda = 1,2,3$.

$$\left.\begin{matrix} \Gamma_{\underset{\vee}{12}}, \Gamma_{\underset{\vee}{13}}, \Gamma_{\underset{\vee}{23}} \\ \Gamma_{\underset{\vee}{21}}, \Gamma_{\underset{\vee}{31}}, \Gamma_{\underset{\vee}{32}} \end{matrix}\right\} \text{Represent the magnetic stress.}$$

In the Generalized Riemannian Geometry Equation of Motion of a Charged Particle in the Electromagnetic Field

To find the equation of motion of a charged particle in electromagnetic field, we use Lorentz force

$$\mathsf{F} = q(\mathsf{E} + v \times \mathsf{B})$$

Where F is the force, q is the charge, v is the velocity and E and B are electric and magnetic fields.

The above equation can be written as

$$\mathsf{F} = q(\mathsf{E} + v \times \mathsf{B})$$

Where F, E and B are weighted tensor.

Taking the force about x ax is we get

$$\begin{aligned}\frac{d\mathsf{P}'}{dt} &= q(\mathsf{E}x' + [v \times \mathsf{B}]x') \\ &= q(\mathsf{E}x' + v_y\mathsf{B}_z - v_z\mathsf{B}_y) \\ &= q\left(v_0 \underset{\vee}{g}^{10} + v_1 \underset{\vee}{g}^{11} + v_2 \underset{\vee}{g}^{12} + v_3 \underset{\vee}{g}^{13} \right)\end{aligned}$$

Or

$$\frac{dP'}{dt} = qv_\beta \underset{\vee}{\mathsf{g}}^{*1\beta} \qquad \beta = 0,1,2,3$$

Generalizing it, we get

$$\frac{d}{dt}\mathsf{P}^\alpha = qv_\beta \underset{\vee}{\mathsf{g}}^{*\alpha\beta}$$

Conclusion

The transformations $\underset{\vee}{g}_{ij} \to \underset{\vee}{\mathsf{g}}_{ij}$ and $\underset{\vee}{\mathsf{g}}_{ij} \to \underset{\vee}{\mathsf{g}}^{*ij}$ are bijective. Hence a direct transformation $\underset{\vee}{g}_{ij} \to \underset{\vee}{\mathsf{g}}^{*ij}$ can be obtained.

Since $\underset{\vee}{\mathsf{g}}^{*ij}$ represent the force similar to Maxwell force, the difference is that of values only. Hence in generalized Riemannian geometry the Maxwell equations will be of the form

$$\frac{\partial \underset{\vee}{g}^{*\mu\nu}}{\partial x^\nu} = \mathsf{J}^\mu$$

And

$$\frac{\partial \underset{\vee}{g}^{*\mu\nu}}{\partial x^\sigma} + \frac{\partial \underset{\vee}{g}^{*\nu\sigma}}{\partial x^\mu} + \frac{\partial \underset{\vee}{g}^{*\sigma\mu}}{\partial x^\nu} = 0$$

According to Carton [4], in the presence of torsion, the Ricci tensor is antisymmetric. Hence by splitting the asymmetric tensors in the sum of symmetric and antisymmetric parts, and then separating symmetric and antisymmetric equations. We get equations of gravitational field and electromagnetic field. Using Lorentz force we get the equation of motion in generalized Riemannian geometry which contains the term $\mathsf{g}^{*ij}_{\;\;v}$ representing the force.

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