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ON THE SURD EQUATION $A\sqrt[a]{x} + B\sqrt[b]{y} = C\sqrt[c]{z}, (a,b,c \in Q)$

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ABSTRACT

In this paper, non-zero integer solutions to three special transcendental equations in surds represented by $\sqrt[p]{x^{3}} + \sqrt[q]{y^{3}} = 2\sqrt[r]{z^{4}} (p,q \succ 3, r \succ 4)$, $\sqrt[p]{x^{3}} + \sqrt[q]{y^{3}} = 2\sqrt[r]{z^{5}} (p,q \succ 3, r \succ 5)$ and $P\sqrt[p]{x^{2}} + Q\sqrt[q]{y^{2}} = (P+Q)\sqrt[r]{z^{2}} (p,q,r \succ 2)$ are obtained.

INTRODUCTION

Dipophantine equations have an unlimited field of research by reason of their variety. Most of the Diophantine problems are algebraic equation [1,2,3]. It seems that much work has not been done to obtain integral solutions of transcendental equations. In this context one may refer [4-19]. This communication analysis a transcendental equation given by $A\sqrt[a]{x} + B\sqrt[b]{y} = C\sqrt[c]{z}, (a,b,c \in Q)$.

Method of Analysis

Surd Equation I:

$$\sqrt[p]{x^{3}} + \sqrt[q]{y^{3}} = 2\sqrt[r]{z^{4}} (p,q \succ 3, r \succ 4) \quad \text{..........(1)}$$

Introducing the transformations

$$x = \sim^{p}, y = \;^{q}, z = ¥^{r} \quad \text{..........(2)}$$

in (1), it leads to

$$\sim^{3} + \;^{3} = 2¥^{4} \quad \text{..........(3)}$$

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Taking $\sim\ = u+v, \quad = u-v, ¥ = u$ ……………………………………………………………(4)

in (3), we have

$$u^2 + 3v^2 = u^3$$

which is satisfied by

$$u = 3m^2 + 1, v = m(3m^2 + 1)$$

Substituting the above values of u,v in (4) and using (2), the required values of x,y,z satisfying (1) are given by

$$x = \left[(3m^2 + 1)(1+m)\right]^p$$
$$y = \left[(3m^2 + 1)(1-m)\right]^q$$
$$z = (3m^2 + 1)^r, m \neq 1$$

Surd Equation II:

$$\overline{\sqrt[p]{x}}^{\,3} + \overline{\sqrt[q]{y}}^{\,3} = 2\overline{\sqrt[r]{z}}^{\,5}, \ (p,q \succ 3, r \succ 5)$$ ……………………………………………………(5)

Applying (2) in (5), we get

$$\sim^3 + \quad ^3 = 2¥^5$$ ……………………………………………………(6)

In view of (4),(6) is represented by

$$u^4 - u^2 = 3v^2$$ ……………………………………………………(7)

Assume, $u^2 = 3U^2 + 1$

The above equation is the well known Pellian equation, whose general solution is given by

$$\left.\begin{aligned} u_n &= \frac{1}{2} f_n \\ U_n &= \frac{1}{2\sqrt{3}} g_n \end{aligned}\right\}$$ ……………………………………………………..(8)

Where $f_n = (2+\sqrt{3})^{n+1} + (2-\sqrt{3})^{n+1}$

and $g_n = (2+\sqrt{3})^{n+1} - (2-\sqrt{3})^{n+1}$

Using (8) in (7) and performing a few calculations, we get

$$v_n = u_n U_n = \frac{1}{4\sqrt{3}} f_n g_n$$ ……………………………………………………(9)

Substituting the values of u_n, v_n given by (8) and (9) in (4) and employing (2), the required values of x,y,z satisfying (1) are given by

$$x_n = \left(\frac{1}{2} + \frac{1}{4\sqrt{3}} g_n\right)^p f_n^p$$

$$y_n = \left(\frac{1}{2} - \frac{1}{4\sqrt{3}} g_n\right)^q f_n^q$$

$$z_n = \frac{1}{2^r} f_n^r$$

Surd Equation III:

$$P\sqrt[p]{x}^{\frac{2}{}}+Q\sqrt[q]{y}^{\frac{2}{}}=(P+Q)\sqrt[r]{z}^{\frac{2}{}}\ ,\ (p,q,r\succ 2) \quad \text{...(10)}$$

Applying (2) in (10) ,we have

$$P\sim^{2}+Q\quad^{2}=(P+Q)¥^{2} \quad \text{...(11)}$$

Introduction of the transformations

$$\left.\begin{array}{l}\sim\ =X+QT\\ \quad =X+PT\end{array}\right\} \quad \text{...(12)}$$

in (11) leads to

$$X^{2}+PQT^{2}=¥^{2} \quad \text{...(13)}$$

Case(i)

Let the product PQ be a square-free integer.

In this case, the solutions of (13) are given by

$$T=2rs$$
$$X=PQr^{2}-s^{2}$$
$$¥=PQr^{2}+s^{2}$$

Substituting the values of X, T in (12) and employing (2), the required values of x,y,z satisfying (1) are given by

$$x=\left(PQr^{2}-s^{2}+2Qrs\right)^{p}$$
$$y=\left(PQr^{2}-s^{2}+2\Pr s\right)^{q}$$
$$z=\left(PQr^{2}+s^{2}\right)^{r}$$

Case(ii)

Let the product PQ be a perfect square, say M^{2}.

In this case, (13) is written as

$$X^{2}+(MT)^{2}=¥^{2} \quad \text{...(14)}$$

which is in the form of well-known Pythagorean equation satisfied by

$$X=2M^{2}RS$$
$$T=M(R^{2}-S^{2})$$
$$¥=M^{2}(R^{2}+S^{2}),(R>S>0)$$

Substituting the values of X,T in (12), and employing (2), the required values of x,y,z satisfying (1) are given by

$$x=\left[2M^{2}RS+QM(R^{2}-S^{2})\right]^{p}$$
$$y=\left[2M^{2}RS+PM(R^{2}-S^{2})\right]^{q}$$
$$z=\left[M^{2}(R^{2}+S^{2})\right]^{r}$$

It is worth to note that, (14) is also satisfied by

$X=m^2k^2-s^2$
$T=2ks$
$¥=m^2k^2+s^2 \quad where, k>s>0$

In this case, the corresponding solutions of (1) are given by

$x=(m^2k^2-s^2+2Qks)^p$
$y=(m^2k^2-s^2+2Pks)^q$
$z=(m^2k^2+s^2)^r$

Conclusion

In this paper, we have presented integer solutions to three different Surd equations. To conclude one may attempt to find integer solutions to Surd equations for other choices of A, B,C,a,b,&c in the transcendental equation considered in the title of the paper.

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