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β_1 NEAR SUBTRACTION SEMIGROUPS

[1]*Usha Devi, S. and [2]Jayalakshmi, S.

[1]Research Scholar, Manonmaniam Sundaranar University, Tirunelveli
[2]Associate Professor, Sri Parasakthi College for Women, Courtallam

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ABSTRACT

In this paper we introduce the notation $\beta_1$near-subtraction semigroup and study some of their properties.

INTRODUCTION

Schein (1992) considered systems of the form (X; o;/), where X isa set of functions closed under the composition "o" of functions (and hence (X; o) is a function semigroup) and the set theoretic subtraction "/" (and hence (X;/) is a subtraction algebra in the sense of (Abbott, 1969). He proved that every subtraction semigroup is isomorphic to a difference semigroup of invertible functions. Zelinka (1995) discussed a problem proposed by Schein concerning the structure of multiplication in a subtraction semigroup. He solved the problem for subtraction algebras of a special type, called the atomic subtraction algebras. Jun *et al*. (2007) introduced the notion of ideals in subtraction algebras and discussed characterization of ideals. In (Kim, 2005), Y.B. Jun and H.S.Kim established the ideal generated by a set, and discussed related results. For basic definition one may refer to Pilz (1983). In near rings the notation of β_1 introduced by Sugantha *et al* (2014). Motivated by this concept, we introduced β_1 near subtraction semigroups. (i.e.,) Let X be a right near subtraction semigroup.

Corresponding author: Usha Devi, S.
Research Scholar, Manonmaniam Sundaranar University, Tirunelveli.

If for every x,y in X, xXy =Xxy then we say X is a β_1 near subtraction semigroup. A characterization of β_1 near subtraction semigroup is given. Throughout this paper X stands for a right near subtraction semigroup.

Preliminary Concepts and Results

Definition: 2.1 A nonempty set X together with binary operations " " and is said to be subtraction algebra if it satisfies the following:

(i) x (y x) = x.
(ii) x (x y) = y (y x).
(iii) (x y) z = (x z) – y, for every x, y, z∈ X.

Definition: 2.2 A nonempty set X together with two binary operations " " and "•" is said to be a subtraction emigroup if it satisfies the following:

(i) (X,) is a subtraction algebra.
(ii) (X,•) is a semigroup.
(iii) x(y z) = xy xz and (x y)z = xz yz, for every x, y, z∈ X.

Definition: 2.3 A nonempty set X together with two binary operations " " and '•' is said to be a near subtraction semigroup (right) if it satisfies the following:

(i) (X,) is a subtraction algebra.
(ii) (X,•) is a semigroup.
(iii) (x y)z = xz yz, for every x, y, z∈X.

Remark: 2.4 The symbol X stands for a near subtraction semigroup (X,–,•) with at least two elements. We write xy for x.y for any two elements x, y of X. It is clear that 0.x = 0, for every x∈X. It can be easily proved that x – 0 = x and 0 – x = 0, for all x∈X.

Definition: 2.5

(i) X_0 = {n∈X / n0 = 0} is called the zero-symmetric part of X.
(ii) X_c ={n∈X / n0 = n} = {n∈X / nn′ = n, for all n′∈X } is called the constant part of X.
(iii) X is called zero-symmetric, if X = X_0.
(iv) X is called constant, if X = X_c.
(v) X_d = {n∈X / n(x – y) = nx – ny, for all x, y in X} is the set of all distributive elements of X.
(vi) A near subtraction semigroup X is called distributive, if X = X_d.

Notations: 2.6

(1) E denotes the set of all idempotent of X.
(2) L denotes the set of all nilpotent elements of X.
(3) If A is any non empty subset of X, then A^* = A–{0}.
(4) C(X) denotes the centre of X.
(5) C(a) = {n∈X / an = na}.
(6) X* = X–{0}

Definition: 2.7 A near subtraction semigroup X is saidto be weak commutative if xyz = xzy, for every x,y,z∈X.

Definition: 2.8 An element e∈X is said to be idempotent if e^2 = e.

Definition: 2.9 An element a∈X is said to be central if ax = xa.

Definition: 2.10 An element x∈X is said to be nilpotent if there exists positive integer n such that x^n=0.

Definition: 2.11 X is said to be pseudo commutative if xyz = zyx, for all x,y,z∈X.

Notation: 2.12 If A and B are any two subsets of X, then AB = {ab / a∈A and b∈B} and A∗B = {a(a′–b) – aa′ / a, a′∈A and b∈B}.

Definition: 2.13 A nonempty subset S of a subtraction semigroup X is said to be a subalgebra of X, if x – x′∈S whenever x, x′∈S.

Definition: 2.14 A nonempty subset M of X is called

(i) a left X-subalgebra of X if M is a subalgebra of (X,–) and XM ⊆ M.
(ii) a right X-subalgebra of X if M is a subalgebra of (X,–) and MX ⊆ M.
(iii) an invariant X-subalgebra of X if M is both left and right X-subalgebras of X.
(iv) a sub near subtraction semigroup M of (X,–) is said to be an invariant sub near subtraction semigroup if M is both left and right X-subalgebras of X.

Definition: 2.15 A subtraction semigroup X is said to be IFP (intersection of factors property) if for a,b in X if ab = 0 implies axb = 0, for all x∈X.

Result: 2.16 A near subtraction semigroup X has no non-zero nilpotent elements if and only if $x^2 = 0 \Rightarrow x = 0$, for all x in X.

Definition: 2.17 If X satisfies **(i)** xy = 0 ⇒ yx = 0, for all x,y in X **(ii)** X has IFP then X is said to have (∗, IFP).

Definition: 2.18 A near subtraction semigroup X is regular if for every x in X there is some y in X such that x = xyx.

Remark: 2.19 If L = {0} and X = X_0 then X has (∗,IFP).

On β_1 near subtraction semigroups

In this section, we study some of the important properties of β_1 near subtraction semigroup and give a complete characterization of such near subtraction semigroup.

Definition: 3.1 Let X be a right near subtraction semigroup. If for every x,y in X, xXy =Xxy then we say X is a β_1 near subtraction semigroup.

Example: 3.1.1 Let X={0,a,b,c} in which '–' and '•' is defined as follows

–	0	a	b	c
0	0	0	0	0
a	a	0	a	0
b	b	b	0	0
c	c	b	a	0

•	0	a	b	c
0	0	0	0	0
a	0	0	a	a
b	0	a	c	b
c	0	a	b	c

Then X is a β_1 near subtraction semigroup. But it is not regular, since aba ≠ a.

Example: 3.1.2 Let X={0,1,2,3,4} in which '–' and '•' is defined as follows

–	0	1	2	3	4
0	0	0	0	0	0
1	1	0	1	1	1
2	2	2	0	2	2
3	3	3	3	0	3
4	4	4	4	4	0

•	0	1	2	3	4
0	0	0	0	0	0
1	0	0	4	1	0
2	0	0	3	2	0
3	0	0	2	3	0
4	0	0	1	4	0

Then X is not a β_1 near subtraction semigroup, since 2X2≠X22.

Example: 3.1.3 Let X={0,1,2,3,4,5} in which '–' and '•' is defined as follows.

–	0	1	2	3	4	5
0	0	0	0	0	0	0
1	1	0	3	4	3	1
2	2	5	0	2	5	4
3	3	0	3	0	3	3
4	4	0	0	4	0	4
5	5	5	0	5	5	0

•	0	1	2	3	4	5
0	0	0	0	0	0	0
1	0	4	2	0	4	2
2	0	2	4	0	2	4
3	0	0	0	0	0	0
4	0	4	2	0	4	2
5	0	2	4	0	2	4

Then X is a zero-symmetric β_1 near subtraction semigroup with no identity.

Proposition: 3.2 Let X be a β_1 near subtraction semigroup. If X has identity 1, then X is zero-symmetric.

Proof: Let X be a β_1 near subtraction semigroup. Then for all x,y in X, xXy = Xxy. Putting y=1, we get xX = Xx, for all x in X. When x = 0, 0X = X0 = {0}. It follows that X is zero-symmetric.

Remark: 3.3 The converse of Proposition 3.2 is not valid. For Example, the near subtraction semigroup cited in Example 3.1.3 is a zero-symmetric β_1 near subtraction semigroup, but it has no identity.

Proposition: 3.4 If X is a β_1 near subtraction semigroup then $xXx = Xx^2$, for all x in X.

Proof: Let X is a β_1 near subtraction semigroup. Then by Definition, for all x,y in X, xXy = Xxy……(1). The result follows by replacing y by x in equation (1).

Remark: 3.5 The converse of Proposition 3.4 is not true.

Example: 3.5.1 Let X={0,a,b,c} in which '–' and '•' is defined as follows

–	0	a	b	c
0	0	0	0	0
a	a	0	a	a
b	b	b	0	b
c	c	c	c	0

•	0	a	b	c
0	0	0	0	0
a	0	0	0	a
b	0	a	b	b
c	0	a	b	C

Then X satisfies the condition $xXx = Xx^2$, for all x in X. But it is not a β_1 near subtraction semigroup, since bXc ≠ Xbc.

Proposition: 3.6 Any pseudo commutative near subtraction semigroup with a right identity is weak commutative.

Proof: Let a,b,c∈X and e be a right identity. Then abc = abce = a(bce) = a(ecb) = (ae)cb = acb. This completes the proof.

Proposition: 3.7 Every pseudo commutative near subtraction semigroup with identity is a β_1 near subtraction semigroup.

Proof: Let X be a pseudo commutative near subtraction semigroup. Let x,y∈X. If a∈xXy, then there exists z∈X such that a = xzy = yzx = yxz (by Proposition: 3.6) = zxy. Therefore a∈Xxy. Thus xXy ⊆ Xxy. On the other hand, if b∈Xxy, then for some $x'\in X$, $b = x'yx = xyx' = xx'y$ (by Proposition: 3.6). Consequently, Xxy ⊆ xXy. From these, we get X is a β_1 near subtraction semigroup.

Proposition: 3.8 Homomorphic image of a β_1 near subtraction semigroup is also a β_1 near subtraction semigroup.

Proof: The proof is straight forward.

Proposition: 3.9 Let X be a β_1 near subtraction semigroup. Then X is regular iff $x\in Xx^2$, for all x in X.

Proof: For the only if part, Let X be regular. Then for every x in X, x = xax∈xXx. By Proposition 3.4, $x\in Xx^2$. For the if part, let $x\in Xx^2$, for all x in X. Again by Proposition 3.4, $xXx = Xx^2$. Hence X is regular.

Lemma: 3.10 If xy = 0, for some x,y in X then $(yx)^r = y0$, for every integer r ≥2. If $X = X_0(0)$ then xy = 0 which implies yx = 0 and X has IFP.

Proof: $xy = 0 \Rightarrow (yx)^2 = yx.yx = y0 \Rightarrow (yx)^r = yx.yx. \ldots$ r times (for all integer r ≥2). Also, when $X = X_0(0)$, then $xy = 0 \Rightarrow (yx)^2 = 0 \Rightarrow yx = 0$. Further, for every n∈X, $(xny)^2 = xny.xny = xn(0) = 0$.

Theorem: 3.11 Let X be a zero-symmetric β_1 near subtraction semigroup with regular. Then we have,

(i) L = {0} **(ii)** X has (*, IFP) **(iii)** E ⊆ C(X)

Proof: **(i)** Since X is regular, By Proposition 3.9, $x\in Xx^2$, for all x in X. Therefore $x = ax^2$, for some a in X. Suppose $x^2 = 0$. Clearly then x = 0. By the Result 2.16, we get L = {0}.

(ii) By (i), L={0}. By Remark 2.19, we get X has (*, IFP).

(iii) Let e∈E. Since X is β_1 near subtraction semigroup, eXe = Xe.e = Xe. Therefore for any x in X, exe = ue and xe = eve, for some u,v in X. Now, e(xe) = eve = xe. Thus exe = xe, for all x in X. We also have, (exe – ex)e = 0 which implies e(exe – ex) = 0 which implies ex(exe – ex) = 0 which implies exe(exe – ex) = 0 (by (ii)).

Consequently, $(exe - ex)^2 = 0$ and (i) guarantees exe – ex = 0.Therefore exe = ex, for all x in X. From these, we get ex = xe, for all x in X. Thus E ⊆ C(X).

Lemma: 3.12 If X is regular then **(i)** xa and ax are idempotents. **(ii)** Xx = Xax and (iii) xX = xaX, for every x in X.

Proof: **(i)** By the Definition of regular, we get the result.

(ii) Xx = Xxax ⊆ Xax ⊆ Xx and (iii) follows in a similar fashion.

Theorem: 3.13 Let X be a zero-symmetric near subtraction semigroup with regular. Then X is β_1 iff $xX = Xx^2$, for all x in X and E⊆C(X).

Proof: For 'only if' part, follows from Prposition 3.4 and 3.11(iii).

Conversely, we assume that $xX = Xx^2$, for all x in X and $E \subseteq C(X)$. For any $x \in X$, we have $x = xax \in xX = Xx^2$. Therefore $x = nx^2$, for some n in X. Then $xax = nx^2ax = naxx^2 = axnx^2 = ax^2$ which implies $(xa - ax)x = 0$ (by theorem 3.11(ii)) and $(xa - ax)ax = 0$ which implies $ax(xa - ax) = 0$. Also $(xa - ax)xa = 0$ which implies $xa(xa - ax) = 0$ and $ax(xa - ax) = 0$. Consequently, $(xa - ax)^2 = 0$ and hence $xa = ax$(by Result 2.16). Hence X is regular. Now, $Xxy = (Xx)y = (Xax)y$ (by lemma 3.12) $= (axX)y = (xaX)y = xXy$ (by Lemma 3.12). Hence X is a β_1 near subtraction semigroup.

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REFERENCES

Abbott, J. C. 1969. Sets, Lattices, and Boolean Algebras, Allyn and Bacon, Inc., Boston, Mass.1969.

Dheena, P. and Satheesh Kumar, G. 2007. On strongly regular near subtraction semigroups, commun.Korean Math. Soc. 22, No.3, pp. 323-330.

Jun, Y. B. and Kim, H. S. 2007. On ideals in subtraction algebras, Sci. Math. Jpn. 65, no.1, 129-134.

Kim, K.H. 2005. On Subtraction Semigroups, Scientiae Mathematicae Japanicae 62, no 2, 273-280.

Pilz, 1983. Gunter, Near-rings, North Holland, Amsterdam.

Schein, B. M. 1992. Difference semigroups, Comm. Algebra 20, no. 8, 2153-2169.

Sugantha, G. and Balakrishnan, R. 2014. β_1 Near-Rings, *International Journal of Algebra*, Vol.8, no. 1, 1-7, HIKARI Ltd.

Zelinka, B. 1995. Subtraction semigroups, Math. Bohem. 120, no. 4, 445-447.

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