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# ESTIMATING THE POPULATION MEAN IN SRSWR WITH MINIMUM SAMPLE SIZE AND MINIMUM RELATIVE MEAN SQUARE ERROR

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**ABSTRACT**

This paper is focused on the impact of minimum sample size with minimum relative standard error on the estimation of the population mean in simple random sampling when sampling is with replacement. The data used in this paper is the data of the population of Fish caught by marine recreational Fishermen by species Group and Year of Atlantic and Gulf coasts. To show the above impact Three applications where executed for this purpose. In application one a minimum sample size under SRSWR with minimum relative standard error (RSE) equal to a given values $\phi$ ( 0.30, 0.31 , 0.32 , 0.33 , 0.34 , 0.35 ) is calculated, it has been shown that with every increases in the value of $\phi$ there is a significance decreases equal to one unit approximately in sample size, a 95% confidence interval for the population mean of fish is estimated as$(2042.15\ ,9416.13\ )$, An estimate of total number of fish is 395310.66. In application tow a sample of size $n = 15$units is required to attain 35% relative standard error of the estimator of population mean under SRSWR sampling. In application three we selected a SRSWR sample of fifteen units from the 1999 Washington, D. C population during 1995 in each of the species group selected in the sampleAn estimate of mean number of fish in each species group during 1995 is 5503.60, the sample variance is 55510243.11, and the estimate of the variance of the estimator $\hat{\bar{y}}$ is equal to 3700682.87 A 95% confidence interval for the mean number of fish in each one of the species group caught during 1995 in the hole country is $[1377.23,\quad 9629.97]$, from the confidence interval ranges the estimated mean value is significant.



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# INTRODUCTION

Simple Random Sampling (SRS) is the simple stand most common method of selecting a sample, in which the sample is selected unit by unit, with equal probability of selection for each unitateach draw, Simple Random Sampling (SRS) is the simplest and most common method of selecting a sample, in which the sample is selected unit by unit, with equal l probability of selection for each unit at each draw. In other words, simple random sampling is a method of selecting a sample s of II units from a population n of size N by giving equal probe ability of selection to all units. It is a sampling scheme in which all possible combinations of II units may be formed from the population of N units with the same chance of selection, If a unit is selected, observed, and replaced in the population before the next draw is made and the procedure is repeated n times, it gives rise to a simple random sample of II units. This procedure is known as simple random sampling. with replacement and is denoted as SRSW R(Sarjin Singh). The population is the entire group of subjects the researcher wants information on (Stockemer, 2019); ideally, it is preferable to include whole population to investigate an issue; however, "practically, it is always not possible to study the entire population" (Acharya, 2013, p. 330). The size of the population under consideration is typically taken into account when determining an acceptable sample size. Therefore, we attempt to select a 'sample' that represents the population under study. Researchers will use sampling as a technique (procedure or device) to systematically choose a smaller group of representative objects or people (a subset) from a pre-defined population to act as subjects (data sources) for observation or experimentation in accordance with the goals of their study (Sharma, 2017). Simple random sampling is an extensively used sampling method in scientific research. Simple random sampling is selected for populations which are highly homogenous where the members of the research are randomly selected to participate in the research (Bhardwaj, 2019).

**Population and Sample Mean and Variance:**

We know that the sample mean is

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} \qquad (1)$$

Is an unbiased estimator of the population mean

$$\bar{Y} = \frac{\sum_{i=1}^{N} y_i}{N} \qquad (2)$$

In other word

$$E(\bar{y}) = \bar{Y} \qquad (3)$$

Similarly the estimator $\hat{y}$ is an unbiased estimator of the population total

$$\hat{y} = N\bar{y} = Y \quad (4)$$

**3 -Relative Mean Square Error:** The ratio of standard deviation to population mean is called the coefficient of variation. It is denoted by $C_Y$ that is

$$C_Y = \frac{\sigma_y}{\bar{Y}} \quad (5)$$

It is also called the relative standard error (RSE) , we can write $C_Y$ as

$$C_Y \approx \frac{S_y}{\bar{Y}} \quad (6)$$

We know that, the population variance $\sigma_y^2$ and the second order population moment$S_y^2$ are defined respectively as:

$$\sigma_y^2 = \frac{\sum_{i=1}^{N}(y_i - \bar{Y})^2}{N} = \frac{(N-1)}{N} S_y^2 \quad (7)$$

$$S_y^2 = \frac{\sum_{i=1}^{N}(y_i - \bar{Y})^2}{N-1} (8)$$

The relative mean square error is defined as the square of the coefficient of variation$C_Y$ and is generally written by RMSE.

$$\text{RMSE}= C_y^2 = \frac{\sigma_y^2}{\bar{Y}^2} = \phi^2 \quad (9)$$

**4. *Minimum Sample Size Under SRSWR:*** Using the above equations, the minimum sample size under SRSWR with minimum relative standard error (RSE) equal to a given value $\phi$is

$$n \geq \left[\frac{\sigma_y^2}{\bar{Y}^2 \phi^2}\right] \quad (10)$$

To proof the minimum sample size as in equation(10) we use the followings steps :

$$RSE(\bar{y}) = \sqrt{V(\bar{y})/(E(\bar{y}))^2} = \sqrt{\sigma_y^2/n\bar{Y}^2}$$

We need an unbiased estimator$\bar{y}$ such that

$$RSE(\bar{y}) \leq \phi$$

Or

$$\sigma_y^2/n\bar{Y}^2 \leq \phi^2$$

Or

$$n \geq \left[\frac{\sigma_y^2}{\bar{Y}^2 \phi^2}\right]$$

Hence proofed.

**5. Application (1):** I will use the data of the population of Fish caught by marine recreational Fishermen by species Group and Year of Atlantic and Gulf coasts (Agricultural Statistics (1999 Washington, D. C.)) „ it consists of 69 kinds available at Atlantic and Gulf coasts. From this population, I selected a simple random sample of size 7with replacement (SRSWR), by using a scientific calculator with random number as follows:

46 , 16 , 9 , 22 , 28 , 43 , 49

Now we can estimate the average number of fish in each one of the species group, as well as the total number of fish at Atlantic and Gulf cost during 1995, but before that we must decide the sample size by using the minimum sample size under SRSWR with minimum relative standard error (RSE).We are given , $N = 69$ , $n = 6$ .

**Table 1. Selected sample (SRSWR) of seven species group**

| RAN # | Kind of fish | No. of fish |
|---|---|---|
| 46 | Spot | 11567 |
| 16 | Striped bass | 10758 |
| 9 | Atlantic cod | 1942 |
| 22 | Crevalle jack | 3951 |
| 28 | GraySnaapper | 4552 |
| 43 | Weak fish | 5739 |
| 49 | Black drum | 1595 |

From the sample we can get the followings information:

**Table 2. Sums and square of the selected values**

| Sample Unit | $y_i$ | $y_i^2$ |
|---|---|---|
| 1 | 11567 | 133795489 |
| 2 | 10758 | 115734564 |
| 3 | 1942 | 3771364 |
| 4 | 3951 | 15610401 |
| 5 | 4552 | 20720704 |
| 6 | 5739 | 32936121 |
| 7 | 1595 | 2544025 |
| Σ | 40104 | 325112668 |

Thus the average number of fish in each group is given by

$$\bar{y} = \frac{\sum_{i=1}^{7} y_i}{7} = \frac{40104}{7} = 5729.14$$

A $(1-\alpha)\,100\%$ confidence interval for the population mean is given by:

$$\bar{y} \pm t(\alpha/2, n\text{-}1) * \sqrt{v(\bar{y})} \quad (11)$$

Where

$$v(\bar{y}) = \frac{s_y^2}{n}.$$

Now we have

$$s_y^2 = \frac{1}{n-1}\left\{\sum_{i=1}^{n} y_i^2 - n^{-1}\left(\sum_{i=1}^{n} y_i\right)^2\right\}$$

$$= \frac{1}{6}\left\{325112668 - \frac{(40104)^2}{7}\right\} = 15891853.8$$

Thus

$$v(\bar{y}) = \frac{s_y^2}{n} = \frac{15891853.8}{7} = 2270264.83$$

The 95% confidence interval for the population mean of fish is given by

$$\bar{y} \pm t(\alpha/2, n\text{-}1) * \sqrt{v(\bar{y})} 5729.14 \pm 2.447\sqrt{2270264.83}$$

$$(\ 2042.15\ ,9416.13\ )$$

An estimate of total number of fish is given by
$\hat{y} = N\bar{y} = Y = 69 * 5729.14 = 395310.66$

The 95% confidence interval for the population mean of fish is given by

$N * (\ 2042.15\ ,9416.13\ )\ or\ 69 * (\ 2042.15\ ,9416.13\ )$
(140908.35 , 649712.97)

**5 .Application (2):** To estimate the average or total number of fish in each one of the species groups caught at the Atlantic and Gulf coasts . There are 69 species groups caught during 1995 in the above population of (1999 Washington, D. C.). Now must first decided about the minimum number of the species groups to be selected by SRSWR sampling to get 30% or 40% accuracy of relative standard error. From the given population we have the followings information:

$N = 69, \quad S_y^2 = 37199578, \quad Y = 311528$

$$\bar{Y} = \frac{311528}{69} = 4514.898$$

$$\sigma_y^2 = \frac{(N-1)}{N} S_y^2 = \frac{(69-1)}{69} * 37199578 = 36660453.68$$

For $\phi = 0.35$, we can estimate population total or population mean, the minimum sample size from (10) will be as

$$n \geq \left[\frac{\sigma_y^2}{\bar{Y}^2 \phi^2}\right] = \frac{36660453.68}{0.35^2 * (4514.898)^2} = 14.65 \cong 15$$

The following table shows different sample sizes for different relative standard errors to estimate the population mean

**Table 3. Different sample sizes for different relative standard errors**

| $\phi$ | 0.30 | 0.31 | 0.32 | 0.33 | 0.34 | 0.35 |
|---|---|---|---|---|---|---|
| $n$ | 20 | 19 | 18 | 17 | 16 | 15 |

Which shows that, whenever arelative standard error is increased a sample size will decreases. Thus a sample of size $n = 15$units is required to attain 35% relative standard error of the estimator of population mean under SRSWR sampling.

**6. Application (3):** Now we can select a SRSWR sample of fifteen units from the 1999 Washington, D. C population during 1995 in each of the species group selected in the sample, in order to estimate the average number of fish m and to construct 95% confidence interval for the average number of fish in each species group in the population. By using scientific calculator random number function (RAN#). The random numbers selected from the population with $N = 15$ are:

63 , 16 , 57 , 49 , 18 , 66 , 41 , 35 , 51 , 48 , 25 , 16 , 48 , 36 and 45.

**Table 4. Selected sample (SRSWR) of fifteen species group**

| RAN# | Species group | $y_i$ | $y_i^2$ |
|---|---|---|---|
| 16 | Striped bass | 10758 | 115734564 |
| 16 | Striped bass | 10758 | 115734564 |
| 18 | Black sea bass | 17723 | 314104729 |
| 25 | Florida pompano | 644 | 414736 |
| 35 | White grunt | 5678 | 32239684 |
| 36 | Grunts, other | 3379 | 11417641 |
| 41 | Porgies, other | 484 | 234256 |
| 45 | Silver perch | 2146 | 4605316 |
| 48 | Atlantic croaker | 17753 | 315169009 |
| 48 | Atlantic croaker | 17753 | 315169009 |
| 49 | Black drum | 1595 | 2544025 |
| 51 | Drums, other | 1354 | 1833316 |
| 57 | Little tunny/ Atlbontito | 782 | 611524 |
| 63 | Gulf flounder | 163 | 26569 |
| 66 | Flounders, other | 1284 | 1648656 |
| Σ | | 82554 | 1231487598 |

An estimate of mean number of fish in each species group during 1995 is:

$$\bar{y} = \frac{\sum_{i=1}^{n} y_i}{n} = \frac{82554}{15} = 5503.60$$

Now

$$s_y^2 = \frac{1}{n-1}\left\{\sum_{i=1}^{n} y_i^2 - n^{-1}\left(\sum_{i=1}^{n} y_i\right)^2\right\}$$
$$= \frac{1}{15-1}\left\{1231487598 - \frac{(82554)^2}{15}\right\}$$
$$= 55510243.11$$

And the estimate of the variance of the estimator $\hat{\bar{y}}$ is given by

$$v(\bar{y}) = \frac{s_y^2}{n} = \frac{55510243.11}{15} = 3700682.87$$

A 95% confidence interval for the mean number of fish in each one of the species group caught during 1995 in the hole country is

$\bar{y} \pm t(\alpha/2, n\text{-}1) * \sqrt{v(\bar{y})}$

$5503.6 \mp 2.145 * \sqrt{3700682.87}$

$[1377.23, 9629.97]$

If we compare it with the estimated mean number of fish caughtduring 1995from the source of agricultural statistics (1999) , Washington, D.C. Which is equal to $\bar{Y} = 4514.899$ , it is clear that the class intervals contains the estimated mean value within it is ranges m also from the confidence interval ranges of the estimated mean value is significant.

# DISCUSSION

It is very clear from the above presentation and results, one can concluded that, we must decide the sample size by using the minimum sample size under SRSWR with minimum relative standard error (RSE). Whenever arelative standard error is increased a sample size will decreases, we can estimate population total or mean by using the minimum sample size as,$n = \left[\frac{\sigma_y^2}{\bar{Y}^2 \phi^2}\right]$.And the estimate of the variance of the estimator $\hat{\bar{y}}$ is given by

$$v(\bar{y}) = \frac{s_y^2}{n} = \frac{55510243.11}{15} = 3700682.87$$

A 95% confidence interval for the mean number of fish in each one of the species group caught during 1995 in the hole country is

$[1377.23, 9629.97]$

The confidence interval ranges ofthe estimated mean value is significant.

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