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VIRTUAL LEARNING OBJECTS USED IN THE POLYGONAL CONCEPT OF GEOMETRY IN ELEMENTARY SCHOOL

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ABSTRACT

This qualitative study explores the development of virtual learning objects for mathematics teaching, specifically geometry in elementary school. The study considers the importance of mathematical representation and visualization on screen in the use of digital technologies and the analyses the mathematical arguments presented by elementary school students and their narratives in the discussion of geometric representations in teaching and learning processes. The data collection method sampled students from a public school with the help of filmed sessions, written records, and field notes. The research focused on elementary school because it is a study that presents contributions to the pedagogical practice of teachers who teach mathematics with digital technologies. The organization and analysis of the data included content categories that contemplate the discussion of the concept of a polygonal line and a polygon, the geometric representation and the mathematical argumentation built in that digital interaction. The results indicate that the arguments, narratives, and representations, aspects rescued in the literature presented in this article point to its role in elementary school mathematics learning and are used by the students often show the mathematical implications constructed from students' interaction with virtual objects e their representations on the computer screen.

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INTRODUCTION

This study examines the development of virtual learning objects, representation analysis, and mathematical argumentation in the context of elementary school. It was conducted based on the mathematics classes held in a public elementary school localized on Santa Catarina, state of Brazil. Activities were based on the creation of virtual learning objects with an emphasis on discussion and argumentation in the context of teaching geometry. This qualitative study aims to answer the following question: what are the contributions of virtual learning objects to teaching geometry in elementary school? This question converges to the valorization of argumentation, representation, and visualization supported by the use of digital technologies in mathematics teaching. In this study, the visualization process considers the opinions of Borba and Villarreal (2005), which are understood as a privileged aspect of the computational environment that is often overlooked in the context of mathematics teaching but valued in the construction of geometric concepts. Similarly, we consider the definition of visualization proposed by Zimmermann and Cunningham (1991). These authors describe it as the process of producing or using geometrical or graphical representations of mathematical concepts, principles, or problems that may be hand-drawn or computer-generated. In this sense, the authors and we believe that visualization plays an important role in mathematics teaching and learning, especially in elementary education. Based on this premise, this study investigates and analyzes the argumentation of representations and interpretations of the oral and written narratives of the students participating in activities conducted with virtual learning objects (Scheffer, 2017; Scheffer et al., 2018). Thus, the study considers the fundamental processes of understanding and building geometric concepts as well as teaching and learning skills. It is noteworthy that as the available computerized environments favor teaching and learning processes, they can be better used and should therefore be adopted. They also expand students' reflection on mathematical implications, since they are based on problem-solving aspects applied through teacher–student interaction and the construction of concepts that promote geometric visualization (Cumino et al., 2021). This assumption emphasizes on the need for reflecting on teacher-driven education because a teacher is a professional who should continuously exercise how to learn and critically rethink their practice (Bairral, 2009). In the field of education, information and communication technologies have been found to offer different possibilities for promoting specific knowledge of mathematics, which can be incorporated into pedagogical work. This study addresses the theoretical foundations and discussions of virtual objects in the school context, specifically in mathematics teaching. Further, it focuses on mathematical representation, visualization and argumentation. It concludes by presenting the data used and the corresponding analysis conducted in the study.

A BRIEF REVIEW

Virtual objects applicable in the School context: Digital technologies offer different possibilities for the processing and analysis of information, exploration, experimentation, and problem-solving actions that are regularly observed in mathematics classes. When considering the current social scenario and, by extension, the public education in the state of Santa Catarina, Brazil, it is relevant to consider some documentation, such as the State Curricular Proposal, that supports this system. According to the State Curricular Proposal, the computerized environment present in schools can be further explored and used to enhance teaching, as the exploratory possibilities of these resources are decisive and effective. Assis and Bezerra (2011) point out that the use of software in classrooms must be guided by pedagogical interests; otherwise, this will not lead to a change in the educational process. Gravina and Basso (2012) refer to the variety of resources available to teachers through the use of ICTs, which enables further discussion regarding the inclusion of schools in virtual culture. Conceptions of this nature underline the need for reflection, studies, and research to resize current education by rescuing technological knowledge in the classroom. Accordingly, ideas and concepts make sense by enabling the full construction of knowledge. In an information society, which is highly dependent on technology, education must keep up with such advancements (Löbler et al., 2012; Lopes et al., 2010). Therefore, virtual objects have been integrated into schools using ICTs. As they are interactive resources applied to education, they represent the technological initiatives undertaken for the teaching and learning of several subjects in schools. For Hay and Knaack (2007), virtual learning objects correspond to interactive tools available on the internet that support the learning of specific concepts by enhancing, expanding, or guiding learners' cognitive processes. The conception, a virtual object is considered as an activity when it follows certain principles, such as its reusability in other contexts. For example, other teachers can add more data or elements that are suited to a specific activity before using a virtual object in their class. For the authors (Silva; Cafe; & Catapan, 2010; Battistella & Wangenheim, 2011; Oliveira; Amaral & Domingos, 2011) it is relevant to highlight certain important characteristics of virtual objects. First, they are flexible, as they are simple to construct and can be customized. Second, they can be easily updated, which allows for their customization and interoperability at different times in distinct contexts and platforms. Given these characteristics, the use of virtual objects may contribute to the resignification of pedagogical practice, as the teaching and learning processes may benefit from various languages and new methods (Scheffer et al., 2018). In this sense, by exploring a learning object in the context of searching for knowledge, teachers may find new ways to use it and mediate the original opportunities. Learning objects may contribute significantly to geometric constructions in mathematics, especially when analyzing representations and visualizations on the screens of computers, tablets, and smartphones. They can also contribute to problem-solving and the search for generalizations (Scheffer et al., 2018). From this perspective, various aspects of a learning object, such as its composition, characteristics, structure, and how it should be organized for use in the educational process, should be considered. Consequently, when constructing a virtual object, the interest that the object arouses in its interaction with ICT in the learning process and its respective dynamics should first be acknowledged by educators.

Representation, visualization, and mathematical argumentation in research: One of the principles of mathematical teaching is relating real world observations to representations, principles, and theoretical concepts, as emphasized in the National Curriculum Parameters (Brasil. Secretaria de Educação Fundamental, 1998) and the National Common Curricular Base (Ministério da Educação. Secretaria de Educação Básica, 2017), which value students' opinions. Therefore, the use of different representations (Duval, 1999, 2006) in mathematics teaching enables students to manifest and contribute to the formalization of meaning and mathematical concepts. According to Boavida (2005), argumentation enables students to demonstrate their learning through interactions and records during classes, which mobilizes several forms of reasoning, language, symbols, and images. Therefore, in addition to creating relationships between stakeholders and mobilizing their intentions, strategies, and processes, argumentation also holds merit in a social, scientific, economic, political, and ideological context. Furthermore, argumentation can be observed in various subjects, given its role and importance in the production of scientific knowledge. In this study, we focus on its articulation with logic and linguistics as well as on the development of students' ability to argue, justify, and clarify relationships. From this perspective, mathematical argumentation, which is built through the interaction among ICT activities, is a decisive aspect helping students to learn and evaluate their comprehension (Scheffer, 2017). Boavida (2005) argues that a favorable context for mathematical argumentation constitutes the exploration of situations that seek to obtain mathematically grounded consensus from students, that is, situations that can be triggered by the exploration of tasks that lead to various resolution processes and promote reflection.

Thus, at the time of resolution, the activities developed in computerized environments contribute to mathematical argumentation that explores different situations and values student reflection. It should be highlighted that students' argumentation is obtained through written records and speech during the development of mathematical construction activities on the computer screen, which are both used to solve problems and answer questions. According to Granell (2003), "mathematical knowledge is deeply dependent on a specific language, of a formal character, which differs greatly from natural languages." This gives mathematical language a high degree of generalization, thereby expanding the possibilities for creating new knowledge. Mathematical language involves the translation of natural language into a formalized universal language, thus allowing for the abstraction of the essence of the mathematical relations involved. As for written representation, Powell and Bairral (2006) emphasize that it contributes to the acquisition of a rich vocabulary and to the students' own mathematical understanding. They also point out that if students write about the mathematics they are producing, a broader range of goals may be achieved. Therefore, writing enables students and teachers to improve the development of mathematical thinking. A favorable space for the construction of mathematical argumentation is established in a learning environment, with virtual objects that involve students' and teachers' interactions with ICTs as well as the representations and visualizations when they attribute mathematical meanings. Such spaces also involve the symbolization and analysis of representations. Moreover, oral discourse, as emphasized by Erath (2021), is used and defined in diverse ways in mathematics education, depending on a wider context, learning objectives, and linguistic reference points. Thus, the role of argumentation assumes its importance in school and math learning, mainly when it refers to geometric representation.

In addition, when referring to mathematics and visualization, Palais (1999) states:

> The advantage of supplementing these and other such classic representations of mathematical objects by computer-generated images is not only that a computer allows one to produce such static displays quickly and easily, but in addition it then becomes straightforward to create rotation and morphing animations that can bring the known mathematical landscape to life in unprecedented ways. Even more exciting for the research mathematician are the possibilities that now exist to use mathematical visualization software to obtain fresh insights concerning complex and poorly understood mathematical objects. (p. 647)

Palais refers to visualization as an important point in the representations of images that occur on a computer screen with dynamic movements, rotation, and animations that present the understanding of mathematical objects, mainly through software.

Concerning visualization in teaching and learning, Presmeg (2006) states the following:

> The visual depiction clearly manifests the difference between various types of algebraic word problems, and this visual process is encouraged and enhanced by the dynamic software. (p. 226)

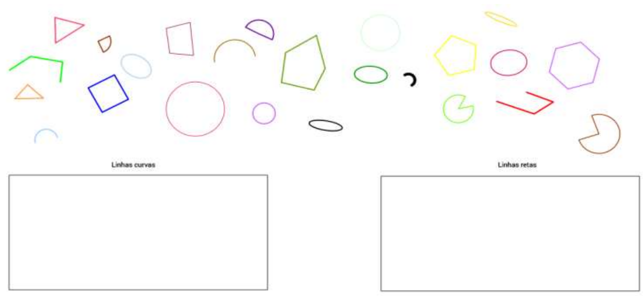
Therefore, similar to Palais (1999), Presmeg (2006) points out that the role of visualization in mathematical teaching and learning perform algebraic relations of various types of mathematical problems in a visual process through dynamic software. Regarding the role of teachers, Presmeg (2006) emphasizes that they can use the visual effects promoted by the software to make connections with other areas of knowledge and relate visualizations to the real world, prompting students to learn mathematical modeling. Presmeg also considers the relationship between algebraic work and conscious attempts by teachers to facilitate students' knowledge construction and focus on the use of imagery. In relation to algebraic work and mathematics argumentation, according to the cited authors, the students resolve to pictorial representations and the use of imagery components through arm, finger, or body movements in the classroom. Teaching with manipulative materials, for example, includes the use of pattern-seeking methods that encourage students' intuition and imagination. These aspects are important in math symbolism, creation, and cognitive construction. Zimmermann and Cunningham (1991) define mathematical visualization as a process of image formation built mentally with pencil and paper or with the aid of digital technologies. Visualization is not considered an end in itself but a means to understanding and discovering mathematical concepts. Computer graphics have greatly expanded the scope and power of visualization in every field, especially in education (Cunningham, 1991; Zimmermann & Cunningham, 1991). Consequently, mathematicians seek patterns in numbers, space, science, computers, and imagination. The use of visualization as a term to describe the process of producing or using geometrical or graphical representations of mathematical concepts and its relation to algebraic math work has been considered by different authors, mainly in mathematical learning and various other fields of teaching (Borba & Villareal, 2005; Cunningham, 1991; Presmeg, 2006; Zimmermann & Cunningham, 1991). Mathematical theories explain the relations among patterns, functions, maps, operators, and transformations on a computer screen or through hand-drawn images. In this sense, if mathematics is considered the science of patterns, it is natural to try to find the most effective ways to visualize these patterns creatively as a tool for understanding. This is the essence of mathematical visualization.

METHODOLOGY OF THE STUDY

This qualitative study was conducted on students from a public school located in the western region of Santa Catarina State, Brazil. It reflects on the presence of virtual environments in the school context by focusing on the importance and interaction of digital technologies in teaching situations to identify, verify, and analyze the mathematical arguments constructed by students regarding geometrical representations. The data collection included filmed sessions during the development of activities involving virtual objects. This methodological procedure, according to Powell and Bairral (2006), enables activities to be viewed and reviewed in the future as often as necessary, thereby enhancing the process of data interpretation. Data collection was also conducted via field notes, observations, and records resulting from the discussions and descriptions of mathematical narratives as well as the arguments and representations built by students. The data were organized by category. According to Franco (2008), this operation classifies the elements in a set by differentiation, which is followed by regrouping based on analogies and previously defined criteria (Bardin, 2016). The data treatment and analysis turns to consider the arguments constructed by the students at the time of interpretation established for the different representations, the forms of interpretation considered from oral expression, gesture, mathematical writing and the figures constructed by them and by the learning objects that the sequence of activities provided took into account the students' mathematical representations and arguments to evaluate different oral and written expressions and math representations, based on the constructions developed by using learning objects. The results of the analysis are briefly discussed in the following section, based on the presentation and description of two activities developed through the construction of learning objects, using GeoGebra software. The data are presented in a matrix of meanings.

Classification of polygonal lines: The tasks involved were a part of a set of activities that aimed to discuss the concept of a polygon by analyzing straight and curved lines as well as open and closed lines. In these activities, the students were asked about the concept and classification of lines and polygons.

Activity 1: Classification of curved or straight lines

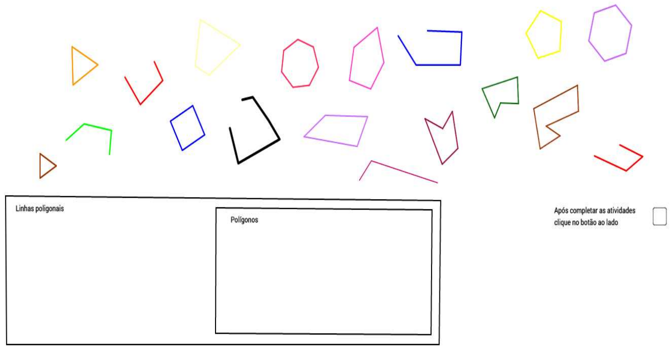


Source: Prepared by the authors, based on the GeoGebra software

Fig. 1. Classification of polygonal lines

In this first activity, different open and closed, curved and straight, and consecutive and non-consecutive lines were presented. The purpose of this task was to select and classify lines into two groups: curved lines and straight lines. Activity 1 required the students to recognize the differences between the presented lines, promoting reflection on the fundamentals that define curved and straight lines.

Activity 2: Classification of open and closed lines



Source: Prepared by the authors, based on the GeoGebra software

Fig. 2. Sorting polygonal lines

In the second activity, only straight open and closed lines were presented. These lines had to be grouped according to the classification criteria of open or closed lines. The students grouped the lines by observing the characteristics of open or closed lines, which constitutes a polygon. After grouping the lines, the students found the definition of a polygon by clicking a button at the right-hand side of the software screen (Fig. 3). Accordingly, they checked the answers and confirmed the concept of the constructed polygon.

After completing the activities, click on the button on the side.
The word polygon comes from the Greek, which means:
Polygon = poly (many) + gon (angles)
Polygons are formed by closed polygonal lines made up of straight line segments.

Source: Prepared by the authors, based on the GeoGebra software

Fig. 3. Hidden text on button click

Empirical insights: In the post-interaction stage, with the learning objects, exploratory questions were proposed regarding the concepts that were worked and organized in an explanatory manner, as shown in Table 1. The analysis of the written records are denoted by A1 through A12, which stand for answer 1, answer 2, answer 3, and so on; they correspond to the contribution of the 12 students who participated in the data collection.

In relation to the representations and mathematics argumentations presented in Table 2, the students' arguments in the classroom included pictorial representations and the use of imagery components through their arm, finger, or body movements. Such movements included the use of pattern-seeking methods, encouraging students' intuition and imagination.

Categorization Categorization involved the classification of constitutive elements of a concept through differentiation and regrouping based on analogies and arguments explored by the participants in the sample. The categories are presented based on a descriptive and analytical approach, referring to the contributions shown in Tables 1 and 2.

Concept of polygonal and polygon lines: Regarding the data on the curved lines presented in Tables 1 and 2—for example, "*it has to be in the shape of a circle"*—we can recognize that the classification made in Activity 1 (Fig. 1) enabled students to identify some common characteristics of the lines selected as curves. Regarding the concept of a polygonal line and a polygon, the answers presented in Activity 2 (Fig. 2)—for example, "*polygonal lines are lines that do not close* or *figures that have an opening"*—show that this classification contributed to the observation of the main characteristics of the lines, which was the activity goal. Such knowledge was confirmed after the interaction with the learning objects, considering that the research participants were senior students who had previously worked on geometric concepts in school. The resolution of geometric problems may give students a better understanding of concepts, as the visualization and representation of geometric concepts are effective ways to construct and explore abstract concepts and establish a relationship between these concepts and reality (Borges & Oliveira, 2016).

Geometric representation: Regarding the classification of closed polygonal lines, that is, polygons presented by the students in their answers—such as "*they are closed polygonal lines* and *they are*

Table 1. Organization and interpretation of data

Categories	*Records and arguments*	*Some interpretations*
Curved line	*A1: "It has no corners."* *A2: "A picture that forms a length (line) that makes a turn (curve) in any direction."* *A3: "It has to be in the shape of a circle."*	*A1: The concept of the curved line has not yet been built.* *A2: It refers to the idea of turning in one direction.* *A3: It associates the idea of a curve with a circle.*
Straight line	*A4: "A straight line is a line that has no curve."* *A5: "It is a flat line."* *A6: "A picture that forms a length in a single direction."*	*A4: It makes a comparison between what is curved and what is straight.* *A5: It relates to a flat line.* *A6: It relates to the idea of infinite measurement and direction.*
Polygonal line	*A7: "Pictures that have an opening"* *A8: "Polygonal lines are lines that do not close."* *A9: "A closed polygonal line is a complete geometric shape."*	*A7: It expresses the understanding of an open line.* *A8: It expresses the understanding of an open line.* *A9: It refers to the idea of polygon as a closed line.*
Polygon	*A10: "They are complete geometric shapes."* *A11: "These are closed polygonal lines."* *A12: "They are formed by closed polygonal lines consisting of straight segments."*	*A10: Complete forms must lead to the interpretation of the polygon by the student.* *A11: It expresses the understanding of closed polygonal lines.* *A12: It gives the definition of a polygon.*

Source: Prepared by the authors, based on the GeoGebra software

Table 2. Arguments and records regarding straight and curved lines

Straight line: some contributions	Curved line: some contributions	Curved and straight lines: some comparisons
"Straight is when the line has no curve; it is super straight."	"Curved lines are circular lines."	"Not every line is the same."
"Straight lines are like a rule."	"Curved lines are curved lines that meet one another."	"A curved line is a 'round' line. A straight line is when there is no curve."
"A straight line occurs when there is no rounded line."		"A curved line is curved, round. A straight line is when there is no rounded line."

Source: Elaborated by the authors, based on the research data

During the development of the two illustrated activities, the students classified the open and closed polygonal lines as well as the curved and straight lines while making intuitive observations relating to the concepts. They demonstrated their comprehension, sense of observation of the details, and characterization of the lines. Such aspects demonstrated by the students can be confirmed by Cunningham (1991) and Zimmermann and Cunningham (1991), who refer to visualization as the process of producing or using geometrical or graphical representations of mathematical concepts in teaching and learning processes. Considering the records written by the students, their interaction with the learning objects influenced their discussion and reflection on geometric aspects, as observed in Table 2. As observed in Table 2, the students' argumentations showed good interaction with the learning objects. As Presmeg (2006) emphasizes, teachers can use the visual effects promoted by software to make connections with other areas. It is crucial to relate the visualizations with representations presented on a computer screen to develop mathematics learning.

complete geometric shapes"—it seems that the activity with the learning objects may have contributed to this observation. Therefore, the lines grouped by different characteristics were decisive in the discussion of the described concept, as the use of words such as "closed" and "complete" refers to the concept of a closed polygonal line and, likewise, to a polygon. The concept "representative" is perceived as an image or the reproduction of an object. When something is revealed and shown, it is as a different reproduction by each one. This constitutes the process of representation associated with the process of visualization (Nemirovsky & Noble, 1997). Based on the exposed activities, a discussion of the polygon concept through the analysis of straight and curved lines and open and closed lines, using learning objects, was successfully developed. The differentiation of open polygonal lines and polygons, as well as the definition of a polygon formed by closed polygonal lines, were therefore decisive aspects for the discussion, reflection, and construction of the concepts involved.

Mathematical argumentation: Regarding the data on the curved lines presented in Tables 1 and 2 — for example, "*it has to be in the shape of a circle",* "Straight is when the line has no curve; it is super straight." — we can recognize that the mathematical argumentation of the students express a classification made in Activity 1 (Fig. 1) to identify some characteristics of the lines selected as curves or straight. The concept of a polygonal line and a polygon, the answers presented in Activity 2 (Fig. 2) — for example, "*polygonal lines are lines that do not close* or *figures that have an opening" or "A straight line occurs when there is no rounded line."* — show that the mathematical argumentation of the students is this classification contributed to the observation of the main characteristics of the lines, which was the activity goal, that were decisive at construction of concepts s straight line and curve line. Finally, the mathematical argumentation is as the teaching and learning processes may benefit from various languages and new methods (Scheffer et al., 2018)

FINAL CONSIDERATIONS

This study included the construction of 10 virtual objects related to geometry, developed to teach regular polygons; it considered the following aspects: straight and curved lines, open and closed lines, consecutive and non-consecutive segments, elements of a polygon (side, angle, and vertex), polygonal lines and polygons, and concurrent and non-concurrent sides and angles. In this study, we focused on concept formation processes in the geometry of polygons built by students. Mathematical concepts are usually developed based on empirical contexts (Dilling & Witzke, 2020). Drawings and the application of virtual objects were used within this empirical context in the study. Learning moments were provided through the analysis of oral arguments and representations, built around the exploration of mathematical objects on a computer screen. Similarly, the use of virtual objects contributed to the resignification of pedagogical practice in a school context, thus, favoring knowledge building. As pointed out by Erath (2021), many studies refer to whole-class discussions or teacher-led group work, emphasizing the crucial role of teachers' support in interaction. The results of the present study show that the use of learning objects can be operationally and effectively considered to support the visualization and characterization of geometric aspects (Scheffer et al., 2018). Learning objects also contribute to the discussion of concepts and properties understood through the dynamism promoted on the computer screen. The contribution of virtual objects was decisive, teachers can use visualization with representations presented on the computer screen through for achieving the activity, as Presmeg (2006) emphasizes. The arguments concerning the representations and interpretations explained by the students are in accordance with Scheffer et al. (2010) in terms of the use of different expressions, manifestations, and interpretations when developing mathematical meanings. As described in Figs. 1 and 2 and discussed in Tables 1 and 2, the qualitative importance of interaction is highlighted when using digital technologies, representations, visualizations, and oral argumentations in mathematical teaching. In addition, the activities conducted in groups were effective for learning, as they were experienced by the students in an information exchange environment, thereby promoting knowledge building. The final results show that most of the time, learning objects serve as a foundation for the construction of mathematical meanings through interaction, dialog, and representations on the computer screen, as developed by students. Regarding polygons, the lines grouped by different characteristics on the computer screen were decisive in the discussion of the concepts described, mainly by referring to the concept of a closed polygonal line, and this is similarly observed in the case of a polygon.

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